Teaching Notes For Homework #7

Fractions

How to say:

$\frac{1}{3}$ = one-third $\frac{3}{4}$ = three-fourths $\frac{1}{2}$ = one-half

A fraction is how much out the total, a part of the whole is.
Sample problem: You have a bag of 31 marbles. Six are green. What fraction of the marbles are NOT green?

Prime and Composite

Prime – a number that can't be divided by a number other than 1 or itself
Composite – a number that can be divided by a number other than 1 or itself
Prime factorization – the final result after doing a tree on a number

Sample problem: 20 - It's composite and it's prime factorization is $2 \bullet 2 \bullet 5$

How do you know when to stop trying to break a number up?
Go through all the divisibility rules, but if the number you are trying times itself is bigger than your number you can stop.

Sample problem 53 - You check to see if 2, 3, and 5 work and they don't. You try 7 and it doesn't work but since 7(7) is 49 we need to keep trying numbers. So we try 11 and again it doesn't work. Since 11(11) is 121 and 121 is larger than 53 we can stop trying numbers and we know for sure that 53 is prime.

Practice Prime and Composite Problems:

- 73
- 114
- 1331
- 51

War Game

Used when you are multiplying and dividing fractions. Also, used on fractions when they are all by themselves to simplify.
War Games are very simple to do and the rules are easy: do trees on everything and then have the army of the north (the top) "fight" the army of the south (the bottom). As long as the entire problem has been converted to all multiplication, you can merge all of the individual fraction bars into one long bar. Once you have finished all of the trees, any of the "soldiers" on the top can fight and "kill" any of the "soldiers" on the bottom as long as they look exactly the same. When the battle is over and the smoke settles and the dust clears, whoever is left alive is your final answer. Keep in mind that if everyone dies on either the top or the bottom, there is

always a one remaining because there are unwritten ones in every multiplication problem that remain even when everything else gets wiped out.

Example: Simplify $\frac{48x^2ya^3}{64xy^2a^2} \div \frac{54x^3y^2a}{72x^2y^2a^2}$

Solution: First convert any divisions to multiplications by flipping over the fraction directly after the division sign to get $\frac{48x^2ya^3}{64xy^2a^2} \cdot \frac{72x^2y^2a^2}{54x^3y^2a}$. You then do trees on everything and write it all out with one fraction bar to get $\frac{2\cdot2\cdot2\cdot2\cdot3\cdot x\cdot x\cdot y\cdot a\cdot a\cdot a\cdot2\cdot2\cdot2\cdot3\cdot3\cdot x\cdot x\cdot y\cdot y\cdot a\cdot a}{2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot x\cdot y\cdot y\cdot a\cdot a\cdot2\cdot3\cdot3\cdot3\cdot x\cdot x\cdot x\cdot y\cdot y\cdot a}$. Now have each matching pair "fight" and "kill" each other by crossing out numbers and letters that match on the top and bottom until there is nothing left on the top that matches anything on the bottom. Keep in mind that the matching pairs do not have to be directly on top of each other for them to fight.

When the war is over, all that remains alive is $\frac{a^2}{y}$.

Sample problem: $\frac{24}{18} \bullet \frac{54}{72} \bullet \frac{64}{48}$

1. Do trees.
2. Kill stuff.

$$\frac{2\bullet2\bullet2\bullet3\bullet3\bullet3\bullet3\bullet2\bullet2\bullet2\bullet2\bullet2\bullet2\bullet2}{2\bullet3\bullet3\bullet2\bullet2\bullet2\bullet3\bullet3\bullet2\bullet2\bullet2\bullet2\bullet3} \rightarrow \frac{2\bullet2}{3} \rightarrow \frac{4}{3}$$

Do not write final answers as mixed numbers. Leave as improper fractions.

Practice War Game Problems:

- $\frac{196}{504}$
- $\frac{48}{72} \bullet \frac{90}{54}$
- $\frac{12x^2ya^3}{20xy^2a^2} \bullet \frac{15x^3y^2a}{18x^2ya^2}$ Show students that the rules still apply even when we put letters in there.
- $\frac{294}{686}$

Introduction to Jealousy Games

This won't be on the homework this week but since this week's material is pretty easy for most students this is a great time to introduce a more complicated one. Having students hear this story multiple times really helps for it to stick into their heads. The Jealousy Game makes the addition and subtraction of basic math and complex, algebraic fractions memorable and easy. Remember, you only play a Jealousy Game when you are adding or subtracting fractions. The story behind this game goes something like this: Each fraction represents a two-story house with a family that lives upstairs and one that lives downstairs. The houses are next door to each other on the same side of the street. Each family has one child and it is your job to babysit the two kids who live downstairs, on the lower level of each house. You manage to babysit both kids at the same time because they are playing with their toys outside in their own front yards. The kids who live upstairs are looking out their front windows, watching the kids you are babysitting play. The toys that the kids are playing with are the numbers and letters that come from doing trees on the bottom of each fraction. The problem is that the kids you are babysitting can see each other's toys and they get jealous and cry because they do not have all the toys that the other has. Since your job is to keep the kids happy, you stop them from crying by giving them the toys they want so that, in the end, all of the kids you are babysitting are playing with exactly the same toys. Mathematically, all of the toys you give to the children are given through multiplication. You think that you have stopped all of the crying, but you are wrong. The kids who live upstairs were watching you give "presents" to the kids who live downstairs so they start crying because it is not fair to give a toy to the downstairs kid and not the upstairs kid. In order to keep the peace, you give each upstairs kid exactly what you gave to the kid you babysat who lived below them. Once all of the kids have stopped crying and are happy, your job is done. Mathematically, the game ends by multiplying together all of the toys that each upstairs kid is playing with and then adding or subtracting those answers together. This becomes the top of the new fraction while the bottom is constructed by simply multiplying one set of toys together from one of the downstairs kids and making that the bottom of this new fraction. The game is now over and the fractions have been combined.

Example: Simplify $\frac{5}{24x^2y}-\frac{1}{18xy^3}$

Solution: Do trees on just the bottoms to determine the toys each downstairs kid is playing with. This gives you $\frac{5}{2\cdot 2\cdot 2\cdot 3\cdot x\cdot x\cdot y}-\frac{1}{2\cdot 3\cdot 3\cdot x\cdot y\cdot y\cdot y}$. In order to stop the jealous crying and to make the downstairs kids happy, you must give a 3 and two y's to the kid on the left and two 2's and an x to the kid on the right. To be fair, you then need to give the upstairs kid on the left the same 3 and two y's that you gave its downstairs kid. Likewise, you also need to give the upstairs kid on the right the same two 2's and an x that you gave to its downstairs kid. This gives you

$$\frac{5\cdot 3\cdot y\cdot y}{2\cdot 2\cdot 2\cdot 3\cdot 3\cdot x\cdot x\cdot y\cdot y\cdot y}-\frac{1\cdot 2\cdot 2\cdot x}{2\cdot 2\cdot 2\cdot 3\cdot 3\cdot x\cdot x\cdot y\cdot y\cdot y}.$$

Multiplying the toys on each top gives you two answers that don't "play" together (cannot be combined through subtraction) so this becomes your new top while the bottom is just one set of toys from a downstairs kid. This gives you the final answer of $\frac{15y^2-4x}{72x^2y^3}$.

Sample problem: $\frac{5}{12}+\frac{1}{18}$

Practice Jealously Game Problems:

- $\frac{7}{48}-\frac{1}{64}$
- $\frac{11}{96}+\frac{19}{162}$