Geometry Homework #21 – Answer Key

1)

Statements	Reasons
$\overline{AB} \perp \overline{AD}, \overline{CD} \perp \overline{DA}$	Given
Angles DAB and ADC are right angles	Perpendicular lines create right angles
$\angle DAB \cong \angle ADC$	All right angles are congruent
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{AD}$	Reflexive
$\Delta DAB \cong \Delta ADC$	SAS
$\angle C \cong \angle B$	CPCTC
Angles BEA & CED are vertical angles	Definition of vertical angles
$\angle BEA \cong \angle CED$	Vertical angles are congruent
$\Delta BEA \cong \Delta CED$	AAS
$\overline{BE} \cong \overline{CE}$	CPCTC

2) Diameter = 22 and the center is $(6,7)$

3)

Statements	Reasons
$\overline{AD} \parallel \overline{BC}$, $\overline{AB} \parallel \overline{DC}$	Given
$\angle BAC \cong \angle DCA, \angle DAC \cong \angle BCA$	If 2 lines are parallel then alternate interior angles are congruent
$\overline{AC} \cong \overline{AC}$	Reflexive
$\Delta ABD \cong \Delta CBE$	SAS
$\angle ABC \cong \angle CDA$	CPCTC

4) Minor axis = 14, major axis = 26, and the center is $(-12,-9)$

5)

Statements	Reasons
$\overline{AB} \parallel \overline{CD}$	Given
$\angle GBE \cong \angle DCE, \angle BGE \cong \angle CDE$	If 2 lines are parallel then alternate interior angles are congruent
Point E is between both C and B & G and D	Definition of vertical angles
$\angle DEC \cong \angle GEB$	All vertical angles are congruent
E is the midpoint of $\overline{CB}$	Given
$\overline{EC} \cong \overline{EB}$	Definition of a midpoint
$\Delta CDE \cong \Delta BGE$	AAS
$\overline{CD} \cong \overline{BG}$	CPCTC
$\overline{AG} \cong \overline{GE}$	Given
$\overline{FC} \perp \overline{AB}$, $\overline{GE} \perp \overline{CB}$	Given
Angles GEB and GAF are right angles	Perpendicular lines create right angles
$\angle GEB \cong \angle GAF$	All right angles are congruent

Angles AGF and EGB are vertical	Definition of vertical angles
$\angle AGF \cong \angle EGB$	Vertical angles are congruent
$\Delta AGF \cong \Delta EGB$	ASA
$\overline{BG} \cong \overline{FG}$	CPCTC
$\overline{CD} \cong \overline{FG}$	Transitive

6) Assume that $\overline{BC} \cong \overline{DA}$. It is given that $\overline{AD} \parallel \overline{CB}$ then $\angle CBD$ and $\angle ADB$ are congruent because parallel lines form congruent, alternate interior angles. $\overline{BD} \cong \overline{BD}$ because of the reflexive property. Therefore, $\Delta CBD \cong \Delta ADB$ because of SAS. Therefore, $\overline{AB} \cong \overline{CD}$ because of CPCTC. But this contradicts the fact that $\overline{AB}$ is not congruent to $\overline{CD}$, which is given. Therefore, the initial assumption must be false. This means that $\overline{BC}$ is not congruent to $\overline{DA}$.

7) Assume that $\angle ADB \cong \angle CBD$. Since angles ADB and CBD are alternate interior angles of $\overline{AD}$ and $\overline{CB}$, and they are congruent, $\overline{AD}$ and $\overline{CB}$ must be parallel because congruent, alternate interior angles form parallel lines. If $\overline{AD} \parallel \overline{CB}$ then $\angle ACB$ and $\angle DAC$ are congruent because parallel lines form congruent, alternate interior angles. But this contradicts the fact that $\angle ABC$ is not congruent to $\angle DAC$, which is given. Therefore, the initial assumption must be false. This means that $\angle ADB$ is not congruent to $\angle CBD$.

8) Assume that B is the midpoint of $\overline{AC}$. Therefore, $\overline{AB} \cong \overline{BC}$ because of the definition of a midpoint. It is given that $\overline{BC} \cong \overline{BD}$. According to the transitive property, $\overline{AB} \cong \overline{BD}$. But this contradicts the fact that $\overline{AB}$ is not congruent to $\overline{BD}$, which is given. Therefore, the initial assumption must be false. This means that B is not the midpoint of $\overline{AC}$.

9) Assume that $\angle ABG \cong \angle EFG$. It is given that $\Delta AGF \cong \Delta EGB$, so $\overline{FG} \cong \overline{BG}$ by CPCTC. Angles FGE and BGA are vertical because of the definition of vertical angles. $\angle FGE \cong \angle BGA$ because vertical angles are congruent. Therefore, $\Delta FGE \cong \Delta BGA$ because of ASA. Therefore, $\overline{BA} \cong \overline{FE}$ because of CPCTC. But this contradicts the fact that $\overline{BA}$ is not congruent to $\overline{FE}$, which is given. Therefore, the initial assumption must be false. This means that $\angle ABG$ is not congruent to $\angle EFG$.

10) Assume that $\overline{BD}$ is an altitude of ΔABC. $\overline{BD} \perp \overline{AC}$ because an altitude of a triangle is always perpendicular to the opposite side. This means that both angles BDC and BDA are right angles because perpendicular lines create right angles. $\angle BDC \cong \angle BDA$ because all right angles are congruent. $\overline{BD} \cong \overline{BD}$ because of the reflexive property and it is given that $\angle A \cong \angle C$. Therefore, $\Delta BCD \cong \Delta BAD$ because of AAS. Therefore, $\overline{AD} \cong \overline{CD}$ because of CPCTC. This means that point D is the midpoint of $\overline{AC}$ because it divides the original segment into two congruent segments. Therefore, $\overline{BD}$ is a median of ΔABC because a median goes through the midpoint. But this contradicts the fact that $\overline{BD}$ is not a median of ΔABC, which is given. Therefore, the initial assumption must be false. This means that $\overline{BD}$ is not an altitude of ΔABC.

11) Assume $\angle BAC \cong \angle FAC$. Angles BCA and ECD are vertical because of the definition of vertical angles. $\angle BCA \cong \angle ECD$ because vertical angles are congruent. It is given that $\angle GCE \cong \angle ECD$. Therefore, $\angle GCE \cong \angle BCA$ by the transitive property. Angles GCE and ACF are vertical because of the definition of vertical angles. $\angle GCE \cong \angle ACF$ because vertical angles are congruent. According to the reflexive property, $\overline{AC} \cong \overline{AC}$. Therefore, $\Delta FCA \cong \Delta BCA$ because of ASA. Therefore, $\overline{FC} \cong \overline{BC}$ because of CPCTC. But this contradicts the fact that $\overline{FC}$ is not congruent to $\overline{BC}$, which is given. Therefore, the initial assumption must be false. This means that $\angle BAC$ is not congruent to $\angle FAC$.

12) Assume that $\angle ABC \cong \angle EDC$. It is given that point C is the midpoint of $\overline{DB}$. Therefore, $\overline{DC} \cong \overline{BC}$ because of the definition of a midpoint. Angles ACB and ECD are vertical because of the definition of vertical angles. $\angle ACB \cong \angle ECD$ because vertical angles are congruent. Therefore, $\Delta BCA \cong \Delta DCE$ because of ASA. Therefore, $\overline{AB} \cong \overline{ED}$ because of CPCTC. But this contradicts the fact that $\overline{AB}$ is not congruent to $\overline{ED}$, which is given. Therefore, the initial assumption must be false. This means that $\angle ABC$ is not congruent to $\angle EDC$.

13) Assume that $\angle B \cong \angle D$. Angles ACB and ECD are vertical because of the definition of vertical angles. $\angle ACB \cong \angle ECD$ because vertical angles are congruent. It is given that $\overline{EC} \cong \overline{AC}$. Therefore, $\Delta ACB \cong \Delta ECD$ because of AAS. Therefore, $\overline{AB} \cong \overline{DE}$ because of CPCTC. But this contradicts the fact that $\overline{AB}$ is not congruent to $\overline{DE}$, which is given. Therefore, the initial assumption must be false. This means that $\angle B$ is not congruent to $\angle D$.

14) Assume that $\overline{BC} \cong \overline{DA}$. It is given that $\overline{AD} \parallel \overline{BC}$ which means that $\angle ACB$ and $\angle CAD$ are congruent because parallel lines form congruent, alternate interior angles. $\overline{AC} \cong \overline{AC}$ because of the reflexive property. Therefore, $\Delta ACB \cong \Delta CAD$ because of SAS. Therefore, $\overline{DC} \cong \overline{BA}$ because of CPCTC. But this contradicts the fact that $\overline{DC}$ is not congruent to $\overline{BA}$, which is given. Therefore, the initial assumption must be false. This means that $\overline{AB}$ is not congruent to $\overline{CD}$.

15) Assume $\overline{FB} \perp \overline{AB}$. This means that $\angle FBA$ is a right angle because perpendicular lines create right angles. Angles ACG and ECD are vertical and so are angles HAC and FAB because of the definition of vertical angles. This means that $\angle DCE \cong \angle GCA$ and $\angle HAC \cong \angle FAB$ because vertical angles are congruent. It is given that $\overline{AB} \parallel \overline{CD}$ which means that $\angle GCA$ and $\angle HAC$ are congruent because parallel lines form congruent, alternate interior angles. By the transitive property, $\angle DCE \cong \angle HAC$ and $\angle DCE \cong \angle FAB$. It is given that $\overline{CE} \cong \overline{AF}$ and that $\overline{AB} \cong \overline{CD}$. Therefore, $\Delta DCE \cong \Delta FAB$ because of SAS. Therefore, $\angle FBA \cong \angle EDC$ because of CPCTC. Since $\angle FBA$ is a right angle, $\angle EDC$ is a right angle by substitution. This makes $\overline{ED} \perp \overline{CD}$ because perpendicular lines are formed by right angles. But this contradicts the fact that $\overline{ED}$ is not perpendicular to $\overline{CD}$, which is given. Therefore, the initial assumption must be false. This means that $\overline{FB}$ is not perpendicular to $\overline{AB}$.

16) Assume that $\overline{AD} \cong \overline{CD}$. Angles FDA and EDC are vertical because of the definition of vertical angles. $\angle FDA \cong \angle EDC$ because vertical angles are congruent. It is given that $\angle A \cong \angle C$. Therefore, $\Delta AFD \cong \Delta CED$ because of ASA. Therefore, $\overline{FD} \cong \overline{ED}$ because of CPCTC. It is given that $\angle FDB \cong \angle EDB$. According to the reflexive property, $\overline{BD} \cong \overline{BD}$. Therefore, $\Delta FDB \cong \Delta EDB$ because of SAS. Therefore, $\overline{FB} \cong \overline{EB}$ because of CPCTC. But this contradicts the fact that $\overline{FB}$ is not congruent to $\overline{EB}$, which is given. Therefore, the initial assumption must be false. This means that $\overline{AD}$ is not congruent to $\overline{CD}$.

17) Assume that $\overline{CD} \cong \overline{ED}$. Angles EDF and CDB are vertical because of the definition of vertical angles. $\angle EDF \cong \angle CDB$ because vertical angles are congruent. It is given that D is the midpoint of $\overline{FB}$. Therefore, $\overline{FD} \cong \overline{BD}$ by the definition of a midpoint. Therefore, $\Delta EDF \cong \Delta CDB$ because of ASA. Therefore, $\overline{CB} \cong \overline{EF}$ because of CPCTC. It is given that $\overline{AE} \cong \overline{CB}$. According to the transitive property, $\overline{AE} \cong \overline{EF}$. But this contradicts the fact that $\overline{AE}$ is not congruent to $\overline{EF}$, which is given. Therefore, the initial assumption must be false. This means that $\overline{CD}$ is not congruent to $\overline{ED}$.

18) Assume that $\angle A \cong \angle EFD$. It is given that $\overline{AB} \parallel \overline{CD}$ which means that $\angle ADE$ and $\angle FED$ are congruent because parallel lines form congruent, alternate interior angles. According to the reflexive property, $\overline{ED} \cong \overline{ED}$. Therefore, $\Delta ADE \cong \Delta FED$ because of AAS. It is given that $\angle EDF \cong \angle DFC$ and the reflexive property guarantees that $\overline{DF} \cong \overline{DF}$. Therefore, $\Delta DCF \cong \Delta FED$ because of ASA. Therefore, $\Delta ADE \cong \Delta DCF$ because of the transitive property. Therefore, $\angle A \cong \angle FDC$ because of CPCTC. It is given that $\overline{AB} \parallel \overline{CD}$ which means that $\angle DCF$ and $\angle EFB$ are congruent and that $\angle A$ and $\angle BEF$ are congruent because parallel lines form congruent, corresponding angles. By the transitive property, $\angle FDC \cong \angle BEF$ According to the reflexive property, $\overline{ED} \cong \overline{ED}$. Therefore, $\Delta BEF \cong \Delta FDC$ because of ASA. Therefore, $\overline{BF} \cong \overline{FC}$ because of CPCTC. But this contradicts the fact that $\overline{FC}$ is not congruent to $\overline{BF}$, which is given. Therefore, the initial assumption must be false. This means that $\angle A$ is not congruent to $\angle EFD$.

19) Assume that $\overline{CE} \cong \overline{BF}$. It is given that $\overline{CD} \cong \overline{BA}$ and $\overline{DE} \cong \overline{AF}$. Therefore, $\Delta ABF \cong \Delta DCE$ because of SSS. Therefore, $\angle BAF \cong \angle CDE$ because of CPCTC. Since angles BAF and CDE are alternate interior angles of $\overline{AB}$ and $\overline{CD}$, and they are congruent, $\overline{AD}$ and $\overline{CB}$ must be parallel because congruent, alternate interior angles form parallel lines. But this contradicts the fact that $\overline{AB}$ is not parallel to $\overline{CD}$, which is given. Therefore, the initial assumption must be false. This means that $\overline{CE}$ is not congruent to $\overline{BF}$.

20) Assume that $\overline{AB} \parallel \overline{CD}$. If $\overline{AB} \parallel \overline{CD}$, which means that $\angle BAC$ and $\angle ECA$ are congruent because parallel lines form congruent, alternate interior angles. It is given that $\overline{AC} \parallel \overline{BD}$ which means that $\angle BAC$ and $\angle FBA$ are congruent because parallel lines form congruent, alternate interior angles. Therefore, $\angle BAC \cong \angle FBA$. According to the transitive property, $\angle FBA \cong \angle ECA$. It is given that $\angle ECA$ is a right angle. Therefore, by substitution, $\angle FBA$ is

a right angle. This means that $\overline{BD} \perp \overline{AC}$ because right angles are formed by perpendicular lines. But this contradicts the fact that $\overline{BD}$ is not perpendicular to $\overline{AB}$, which is given. Therefore, the initial assumption must be false. This means that $\overline{AB}$ is not parallel to $\overline{CD}$.