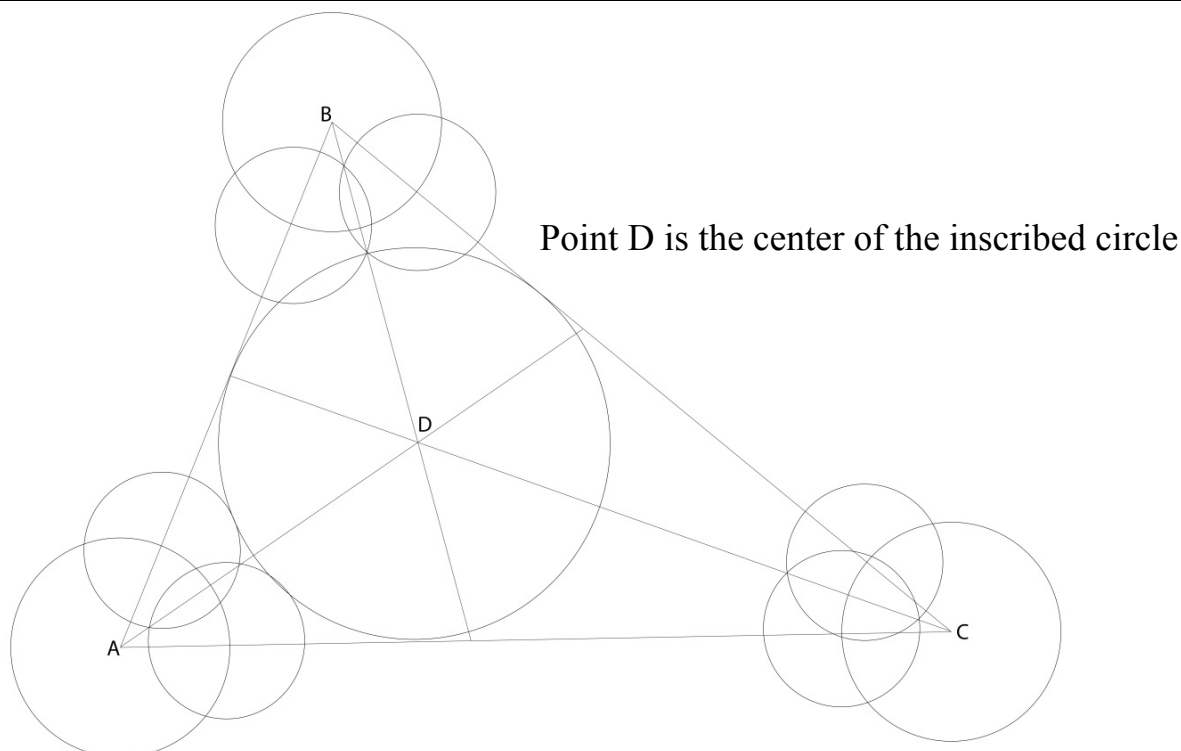


## Geometry Homework #22 – Answer Key

- 1) Assume that  $\overline{AE} \cong \overline{CD}$ . It is given that  $\angle DEH \cong \angle HIJ$ . It is also given that  $\overline{DE} \parallel \overline{PF}$  which means that  $\angle HDE$  and  $\angle HIJ$  are congruent because parallel lines form congruent, alternate interior angles. By the transitive property,  $\angle DEH \cong \angle HDE$ .  $\overline{DE} \cong \overline{DE}$  because of the reflexive property. It is also given that  $\angle A \cong \angle C$ . Therefore,  $\triangle ADE \cong \triangle CED$  because of AAS. Therefore,  $\overline{AE} \cong \overline{CD}$  because of CPCTC. But this contradicts the fact that  $\overline{DC}$  is not congruent to  $\overline{DA}$ , which is given. Therefore, the initial assumption must be false. This means that  $\overline{AE}$  is not congruent to  $\overline{CD}$ .
- 2) Andrew: lions, pretzels, t-shirt Derek: alligators, popcorn, keychain Jacob: giraffe, chips, plastic snake Abigail: monkeys, nachos, stuffed monkey Chloe: bears, crackers, stuffed rabbit
- 3)

Statements	Reasons
$\triangle ACB$ with median $\overline{BD}$	Given
Point D is the midpoint of $\overline{AC}$	Definition of a median
$\overline{AE} \cong \overline{BE}$	Definition of a midpoint
$\triangle ACB$ with altitude $\overline{BD}$	Given
$\overline{BD} \perp \overline{AC}$	Definition of an altitude
Angles $ADB$ & $CDB$ are both right angles	Perpendicular lines form right angles
$\angle ADB \cong \angle CDB$	All right angles are congruent
$\overline{ED} \cong \overline{ED}$	Reflexive
$\triangle AED \cong \triangle CED$	SAS
$\overline{AE} \cong \overline{CE}$	CPCTC
$\angle AEB \cong \angle CEB$	Given
$\overline{BE} \cong \overline{BE}$	Reflexive
$\triangle AEB \cong \triangle CEB$	SAS
$\overline{AB} \cong \overline{CB}$	CPCTC

4)



- 5) Assume  $\overline{AB} \cong \overline{CB}$ . It is given that  $\overline{BD}$  is a median of  $\triangle ABC$ . This means that point D is the midpoint of  $\overline{AC}$  because that is the definition of a median. According to the definition of a midpoint,  $\overline{AD} \cong \overline{CD}$ .  $\overline{DE} \cong \overline{DE}$  because of the reflexive property. Therefore,  $\triangle ADB \cong \triangle CDB$  because of SSS. Therefore,  $\angle BAD \cong \angle BCD$  because of CPCTC. Given that points E, A, D, C, and F are on the same line,  $\angle EAB$  and  $\angle BAD$  are supplementary and  $\angle BCF$  and  $\angle BCD$  because each pair of angles are formed from straight lines. According to the definition of supplementary angles,  $\angle BAD + \angle EAB \cong 180^\circ$  and  $\angle FCB + \angle BCD \cong 180^\circ$ . The transitive property guarantees that  $\angle BAD + \angle EAB \cong \angle FCB + \angle BCD$ . The subtraction property of equality results in  $\angle EAB \cong \angle FCB$ . But this contradicts the fact that  $\angle EAB$  is not congruent to  $\angle ECB$ , which is given. Therefore, the initial assumption must be false. This means that  $\overline{AB}$  is not congruent to  $\overline{CB}$ .
- 6) Given  $-9^0 - 4(-3-2) - 2^3 - 12 \div 4(-2-1) - 5$  and following the mathematical simplification rules known as the order of operations gives us  $-9^0 - 4(-5) - 2^3 - 12 \div 4(-3) - 5$  by simplifying what's inside the parentheses first. Simplifying the exponents next yields  $-1 - 4(-5) - 8 - 12 \div 4(-3) - 5$ . Multiplying and dividing from left to right simplifies the expression to  $-1 + 20 - 8 + 9 - 5$ . Finally, by adding and subtracting from left to right, simplifies the expression to 15.
- 7) Given point A at  $(-22, 4)$ , B at  $(2, -14)$ , C at  $(-11, 12)$ , and D at  $(17, -9)$ , the slope between any two points can be found using the formula  $m = \frac{y_1 - y_2}{x_1 - x_2}$ . For the point A,  $x_1 = -22$  and  $y_1 = 4$  and for point B,  $x_2 = 2$  and  $y_2 = -14$ . Using substitution,  $m = \frac{1(4) - 1(-14)}{1(-22) - 1(2)}$ .
- Following the order of operations and multiplying yields  $m = \frac{4 + 14}{-22 - 2}$ . Finishing the order of operations by adding and subtracting from left to right yields  $m = \frac{18}{-24}$ . Reducing fractions gives a final slope of line AB  $= \frac{3}{-4}$ . In a similar fashion, for the point C,  $x_1 = -11$  and  $y_1 = 12$  and for point D,  $x_2 = 17$  and  $y_2 = -9$ . Using substitution,  $m = \frac{1(12) - 1(-9)}{1(-11) - 1(17)}$ .
- Following the order of operations and multiplying yields  $m = \frac{12 + 9}{-11 - 17}$ . Finishing the order of operations by adding and subtracting from left to right yields  $m = \frac{21}{-28}$ . Reducing fractions gives a final slope of line CD  $= \frac{3}{-4}$ . Since the slopes of lines AB and CD are equal, the two lines must be parallel.
- 8) Given  $-3x - 2^4 - 3(4x - 1) - 24 \div 6(-1 - 3) = -6x - 4(6 - 9)^2 - 2(-5x + 7)$  and following the rules for solving an algebraic equation, you first separate the equation at the equal symbol into

two expressions and follow the mathematical simplification rules known as the order of operations. Simplifying what's inside the parentheses on the left results in  $-3x - 2^4 - 3(4x - 1) - 24 \div 6(-4)$  and on the right results in  $-6x - 4(-3)^2 - 2(-5x + 7)$ . Simplifying the exponents on the left next yields  $-3x - 16 - 3(4x - 1) - 24 \div 6(-4)$  and on the right yields  $-6x - 4(9) - 2(-5x + 7)$ . Multiplying and dividing from left to right simplifies the left expression to  $-3x - 16 - 12x + 3 + 16$  and the right expression to  $-6x - 36 + 10x - 14$ . Finally, by adding and subtracting from left to right, simplifies the left expression to  $-15x + 3$  and the right expression to  $4x - 50$ . You must now use the rules for solving equations to finish solving this problem. Rule 1: If the variable lives on both sides of the equal sign, then one of them must be eliminated. Using the rule that you can add or subtract anything from one side of the equation as long as you do exactly the same thing to the other side, we can add  $15x$  to both sides to get  $19x - 50 = 3$ . Rule 2: Begin to get the variable alone by eliminating the addition or subtraction number on the side containing the variable using the same rule as before. In this case, we will add 50 to both sides to get  $19x = 53$ . Rule 3: Get the variable alone by multiplying or dividing by the number in front of the variable depending on where the number is. If the number is next to the variable, you divide both sides. If the number is below the variable, you multiply both sides. In this case, you would divide both sides by 19 to get the final solution of  $x = \frac{53}{19}$

- 9) Given point A at (2, 2), B at (5, 7), and C at (8, 2), we can calculate the distance between each pair of points by finding the distance between each pair of x's and each pairs of y's, squaring each of these numbers, adding the answers together and doing a square root on the result. For the points A and B, the distance between the x's is 3 while the distance between the y's is 5. Squaring both numbers gives you a 9 and a 25 which, when added together gives you 34. The final step is to square root this answer. Therefore, the distance from A to B is  $\sqrt{34}$ . Repeating this process for the points A and C gives you a distance between the x's of 6 while the distance between the y's is 0. Squaring both numbers gives you a 36 and a 0 which, when added together gives you 36. The final step is to square root this answer. Therefore, the distance from A to C is 6. Repeating this process one last time for the points B and C gives you a distance between the x's of 3 while the distance between the y's is 5. Squaring both numbers gives you a 9 and a 25 which, when added together gives you 34. The final step is to square root this answer. Therefore, the distance from B to C is  $\sqrt{34}$ . Since  $AB = \sqrt{34}$  and  $BC = \sqrt{34}$ ,  $AB = BC$  by the transitive property. Therefore triangle ABC is isosceles because the definition of an isosceles triangle is any triangle with two equal sides.

- 10) Given point A at (16, -1), B at (-8, 8), C at (-6, -20), and D at (3, 4), the slope between any two points can be found using the formula  $m = \frac{y_1 - y_2}{x_1 - x_2}$ . For the point A,  $x_1 = 16$  and  $y_1 = -1$  and for point B,  $x_2 = -8$  and  $y_2 = 8$ . Using substitution,  $m = \frac{1(-1) - 1(8)}{1(16) - 1(-8)}$ . Following

the order of operations and multiplying yields  $m = \frac{-1-8}{16+8}$ . Finishing the order of operations by adding and subtracting from left to right yields  $m = \frac{-9}{24}$ . Reducing fractions gives a final slope of line AB  $= \frac{-3}{8}$ . In a similar fashion, for the point C,  $x_1 = -6$  and  $y_1 = -20$  and for point D,  $x_2 = 3$  and  $y_2 = 4$ . Using substitution,  $m = \frac{1(-20)-1(4)}{1(-6)-1(3)}$ . Following the order of operations and multiplying yields  $m = \frac{-20-4}{-6-3}$ . Finishing the order of operations by adding and subtracting from left to right yields  $m = \frac{-24}{-9}$ . Reducing fractions gives a final slope of line CD  $= \frac{8}{3}$ . Since the slopes of lines AB and CD are negative reciprocals of each other, according to the definition of perpendicular lines, the two lines must be perpendicular.

- 11) Given that point A is (6,10), B is (0,-5), C is (10,-9), and D is (-10,-1), in order to prove that AB is both an altitude and median of triangle DAC then you need to show that AB and DC are perpendicular and that B is the midpoint of DC. Since the slope between any two points can be found using the formula  $m = \frac{y_1 - y_2}{x_1 - x_2}$ . For the point A,  $x_1 = 6$  and  $y_1 = 10$  and for point B,  $x_2 = 0$  and  $y_2 = -5$ . Using substitution,  $m = \frac{1(10)-1(-5)}{1(6)-1(0)}$ . Following the order of operations and multiplying yields  $m = \frac{10+5}{6-0}$ . Finishing the order of operations by adding and subtracting from left to right yields  $m = \frac{15}{6}$ . Reducing fractions gives a final slope of line AB  $= \frac{5}{2}$ . In a similar fashion, for the point D,  $x_1 = -10$  and  $y_1 = -1$  and for point C,  $x_2 = 10$  and  $y_2 = -9$ . Using substitution,  $m = \frac{1(-1)-1(-9)}{1(-10)-1(10)}$ . Following the order of operations and multiplying yields  $m = \frac{-1+9}{-10-10}$ . Finishing the order of operations by adding and subtracting from left to right yields  $m = \frac{8}{-20}$ . Reducing fractions gives a final slope of line DC  $= \frac{2}{-5}$ . Since the slopes of lines AB and DC are negative reciprocals of each other, according to the definition of perpendicular lines, the two lines must be perpendicular.

Therefore AB must be an altitude of triangle DAC because the definition of an altitude is that it goes through a vertex and is perpendicular to the opposite side of the triangle. In order to prove that AB is also a median, then point B must be the midpoint of DC. This can be proved

through the use of the midpoint formula. The midpoint formula states that if  $x = \frac{x_1 + x_2}{2}$

and  $y = \frac{y_1 + y_2}{2}$ , then (x, y) is the midpoint. For points D and C,  $x = \frac{1(-10) + 1(10)}{2}$

which simplifies to  $x = \frac{-10 + 10}{2}$  and becomes  $x = 0$  following the order of operations.

Likewise, for D and C,  $y = \frac{1(-1) + 1(-9)}{2}$  which simplifies to  $y = \frac{-1 - 9}{2}$  and becomes  $y = -5$  following the order of operations. This means that the midpoint of DC is (0, -5). Point B is (0, -5). Therefore, B is the midpoint of DC which makes it a median of triangle DAC according to the definition of a median.

- 12) Given  $-2(-4 - 3)^0 - 8 \div 4(-1 - 1) - 4^2 - \sqrt[3]{64} - 3^4$  and following the mathematical simplification rules known as the order of operations gives us  $-2(-7)^0 - 8 \div 4(-2) - 4^2 - \sqrt[3]{64} - 3^4$  by simplifying what's inside the parentheses first. Simplifying the exponents next yields  $-2(1) - 8 \div 4(-2) - 16 - 4 - 81$ . Multiplying and dividing from left to right simplifies the expression to  $-2 + 4 - 16 - 4 - 81$ . Finally, by adding and subtracting from left to right, simplifies the expression to  $-99$ .

- 13) Given that point A is (-13, -5), B is (-3, 2), C is (-7, -9), and D is (13, -2) we could find the

slopes of line AB and line CD using the formula  $m = \frac{y_1 - y_2}{x_1 - x_2}$ . For the point A,

$x_1 = -13$  and  $y_1 = -5$  and for point B,  $x_2 = -3$  and  $y_2 = 2$ . Using substitution,

$m = \frac{1(-5) - 1(2)}{1(-13) - 1(-3)}$ . Following the order of operations and multiplying yields

$m = \frac{-5 - 2}{-13 + 3}$ . Finishing the order of operations by adding and subtracting from left to right

yields  $m = \frac{-7}{-10}$ . Reducing fractions gives a final slope of line AB =  $\frac{7}{10}$ . In a similar

fashion, for the point C,  $x_1 = -7$  and  $y_1 = -9$  and for point D,  $x_2 = 3$  and  $y_2 = -2$ . Using

substitution,  $m = \frac{1(-9) - 1(-2)}{1(-7) - 1(3)}$ . Following the order of operations and multiplying yields

$m = \frac{-9 + 2}{-7 - 3}$ . Finishing the order of operations by adding and subtracting from left to right

yields  $m = \frac{-7}{-10}$ . Reducing fractions gives a final slope of line CD =  $\frac{7}{10}$ . Since the slopes of

lines AB and CD are equal, the two lines must be parallel. Using line CD as a transversal,

angle ABC and angle DCB must be congruent because if two lines are cut by a transversal, alternate interior angles are congruent.

- 14) Given  $-3(-4x - 5) - 54 \div 6(-2 - 7) - 9x = -3^3 - 3(-6x - 4) - 2(-2 - 1)^3 - \sqrt[3]{343} - 4x$  and following the rules for solving an algebraic equation, you first separate the equation at the equal symbol into two expressions and follow the mathematical simplification rules known as the order of operations. Simplifying what's inside the parentheses on the left results in  $-3(-4x - 5) - 54 \div 6(-9) - 9x$  and on the right results in  $-3^3 - 3(-6x - 4) - 2(-3)^3 - \sqrt[3]{343} - 4x$ . Simplifying the exponents on the left leaves the expression unchanged while simplifying the exponents on the right yields  $-27 - 3(-6x - 4) - 2(-27) - 7 - 4x$ . Multiplying and dividing from left to right simplifies the left expression to  $12x + 15 + 81 - 9x$  and the right expression to  $-27 + 18x + 12 + 54 - 7 - 4x$ . Finally, by adding and subtracting from left to right, simplifies the left expression to  $3x + 96$  and the right expression to  $14x + 32$ . You must now use the rules for solving equations to finish solving this problem. Rule 1: If the variable lives on both sides of the equal sign, then one of them must be eliminated. Using the rule that you can add or subtract anything from one side of the equation as long as you do exactly the same thing to the other side, we can subtract  $3x$  from both sides to get  $96 = 11x + 32$ . Rule 2: Begin to get the variable alone by eliminating the addition or subtraction number on the side containing the variable using the same rule as before. In this case, we will subtract 32 to both sides to get  $64 = 11x$ . Rule 3: Get the variable alone by multiplying or dividing by the number in front of the variable depending on where the number is. If the number is next to the variable, you divide both sides. If the number is below the variable, you multiply both sides. In this case, you would divide both sides by 11 to get the final solution of  $x = \frac{64}{11}$

- 15) Given point A at (2, 4), B at (11, -8), and C at (-10, -5), we can calculate the distance between each pair of points by finding the distance between each pair of x's and each pairs of y's, squaring each of these numbers, adding the answers together and doing a square root on the result. For the points A and B, the distance between the x's is 9 while the distance between the y's is 12. Squaring both numbers gives you an 81 and a 144 which, when added together gives you 225. The final step is to square root this answer. Therefore, the distance from A to B is 15. Repeating this process for the points B and C gives you a distance between the x's of 21 while the distance between the y's is 3. Squaring both numbers gives you a 441 and a 9 which, when added together gives you 500. The final step is to square root this answer. Therefore, the distance from B to C is  $10\sqrt{5}$ . Repeating this process one last time for the points A and C gives you a distance between the x's of 12 while the distance between the y's is 9. Squaring both numbers gives you a 144 and an 81 which, when added together gives you 225. The final step is to square root this answer. Therefore, the distance from A to C is 15. Since  $AB = 15$  and  $AC = 15$ , then  $AB = AC$  by the transitive property. Therefore triangle ABC is isosceles because the definition of an isosceles triangle is any triangle with two equal sides. To prove that a triangle is a right triangle, it must satisfy the Pythagorean Theorem which states that  $a^2 + b^2 = c^2$  where a, b, and c are the lengths of the three sides of the triangle with c being the longest side. For our triangle,  $a = AB$ ,  $b = AC$ , and  $c = BC$ . Using substitution, the formula becomes  $(15)^2 + (15)^2 = (10\sqrt{5})^2$ . Using the order of operations to simplify both sides of the

equation, you would get  $225 + 225 = 500$  by evaluating the exponents. Simplifying the left side by adding yields the equation  $500 = 500$  which is a true statement. Therefore, triangle ABC must be a right triangle.

- 16) Given point A is (-5,3), B is (0,-4), C is (-7,-6), D is (-11,4), E is (4,6), and F is (-2,1) we can calculate the distance between each pair of points in triangle EAB by finding the distance between each pair of x's and each pairs of y's, squaring each of these numbers, adding the answers together and doing a square root on the result. For the points E and A, the distance between the x's is 9 while the distance between the y's is 3. Squaring both numbers gives you an 81 and a 9 which, when added together gives you 90. The final step is to square root this answer. Therefore, the distance from E to A is  $3\sqrt{10}$ . Repeating this process for the points E and B gives you a distance between the x's of 4 while the distance between the y's is 10. Squaring both numbers gives you a 16 and a 100 which, when added together gives you 116. The final step is to square root this answer. Therefore, the distance from E to B is  $2\sqrt{29}$ . Repeating this process one last time for the points A and B gives you a distance between the x's of 5 while the distance between the y's is 7. Squaring both numbers gives you a 25 and a 49 which, when added together gives you 74. The final step is to square root this answer. Therefore, the distance from A to B is  $\sqrt{74}$ . Now repeat this process for each pair of points in triangle DFC by finding the distance between each pair of x's and each pairs of y's, squaring each of these numbers, adding the answers together and doing a square root on the result. For the points D and F, the distance between the x's is 9 while the distance between the y's is 3. Squaring both numbers gives you an 81 and a 9 which, when added together gives you 90. The final step is to square root this answer. Therefore, the distance from D to F is  $3\sqrt{10}$ . Repeating this process for the points D and C gives you a distance between the x's of 4 while the distance between the y's is 10. Squaring both numbers gives you a 16 and a 100 which, when added together gives you 116. The final step is to square root this answer. Therefore, the distance from D to C is  $2\sqrt{29}$ . Repeating this process one last time for the points F and C gives you a distance between the x's of 5 while the distance between the y's is 7. Squaring both numbers gives you a 25 and a 49 which, when added together gives you 74. The final step is to square root this answer. Therefore, the distance from F to C is  $\sqrt{74}$ . Since both sides EB and CD equal  $2\sqrt{29}$ , both sides AE and DF equal  $3\sqrt{10}$ , and both sides AB and FC equal  $\sqrt{74}$ , triangles EAB and DFC are congruent by the SSS property. Therefore angles EAB and DFC are congruent by CPCTC.
- 17) Given  $\sqrt[5]{32} - 3|4 - 6| - 1^7 - 12 \div 3 \div (-3 - 1) - 2^6$  and following the mathematical simplification rules known as the order of operations gives us  $\sqrt[5]{32} - 3|-2| - 1^7 - 12 \div 3 \div (-4) - 2^6$  by simplifying what's inside the parentheses first. Since this expression involves absolute value symbols, they must be simplified next yielding  $\sqrt[5]{32} - 3(2) - 1^7 - 12 \div 3 \div (-4) - 2^6$ . Simplifying the exponents next yields  $2 - 3(2) - 1 - 12 \div 3 \div (-4) - 64$ . Multiplying and dividing from left to right simplifies the expression to  $2 - 6 - 1 + 1 - 64$ . Finally, by adding and subtracting from left to right, simplifies the expression to  $-67$ .

- 18) Given point A is (-2,-10), B is (-12,5), C is (4,7), D is (-8,-1), and E is (-4,-7) we can calculate the areas of triangles by determining the length of the base and the length of the height. The height of any triangle is the length of the segment whose one endpoint is on the base, and perpendicular to the base, while the other endpoint is at the opposite vertex of the triangle. By making a quick sketch, the only two segments that might be perpendicular are AB and CD.

Determine the slopes of line AB and line CD using the using the formula  $m = \frac{y_1 - y_2}{x_1 - x_2}$ . For

the point A,  $x_1 = -2$  and  $y_1 = -10$  and for point B,  $x_2 = -12$  and  $y_2 = 5$ . Using

substitution,  $m = \frac{1(-10) - 1(5)}{1(-2) - 1(-12)}$ . Following the order of operations and multiplying yields

$m = \frac{-10 - 5}{-2 + 12}$ . Finishing the order of operations by adding and subtracting from left to right

yields  $m = \frac{-15}{10}$ . Reducing fractions gives a final slope of line AB  $= \frac{-3}{2}$ . In a similar

fashion, for the point C,  $x_1 = 4$  and  $y_1 = 7$  and for point D,  $x_2 = -8$  and  $y_2 = -1$ . Using

substitution,  $m = \frac{1(7) - 1(-1)}{1(4) - 1(-8)}$ . Following the order of operations and multiplying yields

$m = \frac{7 + 1}{4 + 8}$ . Finishing the order of operations by adding and subtracting from left to right

yields  $m = \frac{8}{12}$ . Reducing fractions gives a final slope of line CD  $= \frac{2}{3}$ . Since the slopes of

lines AB and CD are negative reciprocals, the two lines must be perpendicular as that is the definition of perpendicular lines. According to the definition of the height of a triangle, this makes the segment CD the height of any triangle with a base on AB and a vertex at C. To determine the length of CD we need find the distance between each pair of x's and each pairs of y's, squaring each of these numbers, adding the answers together and doing a square root on the result. For the points C and D, the distance between the x's is 12 while the distance between the y's is 8. Squaring both numbers gives you a 144 and a 64 which, when added together gives you 208. The final step is to square root this answer. Therefore, the distance from C to D is  $4\sqrt{13}$ . To find the area of triangle ABC, we need to find the length of the base, AB. Repeating the same process for the points A and B as we did for points C and D gives you a distance between the x's of 10 while the distance between the y's is 15. Squaring both numbers gives you a 100 and a 225 which, when added together gives you 325. The final step is to square root this answer. Therefore, the distance from A to B is  $5\sqrt{13}$ . Using the formula

for calculating the area of any triangle,  $A = \frac{1}{2}bh$ , with the base, AB, being  $5\sqrt{13}$  and the

height, CD, being  $4\sqrt{13}$  gives the equation  $A = \frac{1}{2}(5\sqrt{13})(4\sqrt{13})$  by substitution. Simplifying the right side of the equation through multiplication yields the area of triangle ABC to be 130. To find the area of triangle AEC, we need to find the length of the base, AE. Repeating the



same process for the points A and E as we did for points A and B gives you a distance between the x's of 2 while the distance between the y's is 3. Squaring both numbers gives you a 4 and a 9 which, when added together gives you 13. The final step is to square root this answer.

Therefore, the distance from A to E is  $\sqrt{13}$ . Using the formula for calculating the area of any triangle,  $A = \frac{1}{2}bh$ , with the base, AE, being  $\sqrt{13}$  and the height, CD, being  $4\sqrt{13}$  gives the equation  $A = \frac{1}{2}(\sqrt{13})(4\sqrt{13})$  by substitution. Simplifying the right side of the equation through multiplication yields the area of triangle AEC to be 26.

- 19) Given that point A is  $(-14, -4)$ , B is  $(-4, 10)$ , C is  $(12, 8)$ , and D is  $(2, -6)$  we could find the slopes of line AB, BC, AD and line CD using the using the formula  $m = \frac{y_1 - y_2}{x_1 - x_2}$ . For the point A,  $x_1 = -14$  and  $y_1 = -4$  and for point B,  $x_2 = -4$  and  $y_2 = 10$ . Using substitution,
- $$m = \frac{1(-4) - 1(10)}{1(-14) - 1(-4)}$$
- Following the order of operations and multiplying yields
- $$m = \frac{-4 - 10}{-14 + 4}$$
- Finishing the order of operations by adding and subtracting from left to right yields  $m = \frac{-14}{-10}$ . Reducing fractions gives a final slope of line AB  $= \frac{7}{5}$ . In a similar fashion, for the point C,  $x_1 = 12$  and  $y_1 = 8$  and for point D,  $x_2 = 2$  and  $y_2 = -6$ . Using substitution,
- $$m = \frac{1(8) - 1(-6)}{1(12) - 1(2)}$$
- Following the order of operations and multiplying yields
- $$m = \frac{8 + 6}{12 - 2}$$
- Finishing the order of operations by adding and subtracting from left to right yields  $m = \frac{14}{10}$ . Reducing fractions gives a final slope of line CD  $= \frac{7}{5}$ . Since the slopes of lines AB and CD are equal, the two lines must be parallel. Using line BD as a transversal, angle ABD and angle CDB must be congruent because if two lines are cut by a transversal, alternate interior angles are congruent. We can now find the slopes of lines BC and AD in the same way. For the point B,  $x_1 = -4$  and  $y_1 = 10$  and for point C,  $x_2 = 12$  and  $y_2 = 8$ .
- Using substitution,  $m = \frac{1(10) - 1(8)}{1(-4) - 1(12)}$ . Following the order of operations and multiplying
- $$m = \frac{10 - 8}{-4 - 12}$$
- Finishing the order of operations by adding and subtracting from left to right yields  $m = \frac{2}{-16}$ . Reducing fractions gives a final slope of line BC  $= \frac{1}{-8}$ . In a similar fashion, for the point A,  $x_1 = -14$  and  $y_1 = -4$  and for point D,  $x_2 = 2$  and  $y_2 = -6$ .

Using substitution,  $m = \frac{1(-4) - 1(-6)}{1(-14) - 1(2)}$ . Following the order of operations and multiplying yields  $m = \frac{-4 + 6}{-14 - 2}$ . Finishing the order of operations by adding and subtracting from left to

right yields  $m = \frac{2}{-16}$ . Reducing fractions gives a final slope of line AD =  $\frac{1}{-8}$ . Since the slopes of lines BC and AD are equal, the two lines must be parallel. Using line BD as a transversal, angle ADB and angle CBD must be congruent because if two lines are cut by a transversal, alternate interior angles are congruent. By the reflexive property segment BD equals segment BD. Therefore, triangle ABD is congruent to triangle CBD by the ASA property. Finally, angle BAD must be congruent to angle DCB by CPCTC.

- 20) Given  $8x - 72 \div 9(-7 - 1) - 3(-5x + 7)^0 - 13x = -5x(-1 - 2)^2 - 4^2 - \sqrt[5]{243} - 3(-7x - 8) - 2^4$  and following the rules for solving an algebraic equation, you first separate the equation at the equal symbol into two expressions and follow the mathematical simplification rules known as the order of operations. Simplifying what's inside the parentheses on the left results in  $8x - 72 \div 9(-8) - 3(-5x + 7)^0 - 13x$  and on the right yields  $-5x(-3)^2 - 4^2 - \sqrt[5]{243} - 3(-7x - 8) - 2^4$ . Simplifying the exponents on the left yields  $8x - 72 \div 9(-8) - 3(1) - 13x$  while simplifying the exponents on the right yields  $-5x(9) - 16 - 3 - 3(-7x - 8) - 16$ . Multiplying and dividing from left to right simplifies the left expression to  $8x + 64 - 3 - 13x$  and the right expression to  $-45x - 16 - 3 + 21x + 24 - 16$ . Finally, by adding and subtracting from left to right, simplifies the left expression to  $-5x + 61$  and the right expression to  $-24x - 11$ . You must now use the rules for solving equations to finish solving this problem. Rule 1: If the variable lives on both sides of the equal sign, then one of them must be eliminated. Using the rule that you can add or subtract anything from one side of the equation as long as you do exactly the same thing to the other side, we can add  $24x$  from both sides to get  $19x + 61 = -11$ . Rule 2: Begin to get the variable alone by eliminating the addition or subtraction number on the side containing the variable using the same rule as before. In this case, we will subtract 61 to both sides to get  $19x = -72$ . Rule 3: Get the variable alone by multiplying or dividing by the number in front of the variable depending on where the number is. If the number is next to the variable, you divide both sides. If the number is below the variable, you multiply both sides. In this case, you would divide both sides by 19 to get the final solution of  $x = \frac{-72}{19}$