Teaching Notes for Geometry

Homework #10

Preparation: Watch videos “Geometric sums”and “Fibonacci.”

Since students are already familiar with athematic sequences and series, making the jump to geometric sequence and series is an easy one.

Provide students with the formula: $a_n = a_1 \bullet r^{n-1}$

1) Find a formula to calculate any term for the geometric sequence 5, 25, 125…
   $5(5)^{n-1}$

2) Find a formula to calculate any term for the geometric sequence 64, 16, 4…
   $64(\frac{1}{4})^{n-1}$

Provide students with the formula: $S_n = \dfrac{a_1(1-r^n)}{1-r}$

3) Find the sum of the geometric series 3 + 9 + 27 +…+ 6,561 without actually adding up all of the numbers.   9,840

4) Find the sum of the geometric series 1+2 + 4 + 8 +…+ 2,048 without actually adding up all of the numbers.   4,095

5) Find the sum of the geometric series 25 + 125 + 625 +…+ 15,625 without actually adding up all of the numbers.   19,525

6) During the summer months, algae blooms on Lake Erie grow geometrically according to the sequence 1, 2, 4, 8, 16.... Currently, 1 acre of water is covered by algae. How much of the lake will be covered by algae after 9 weeks?   512 acres

7) All of the town’s drinking water is stored in a huge tank. The tank is full and it holds 1,024 gallons of water. There is a severe drought and the tank will not be refilled for months so the town has to ration the water in the tank. To ensure that the town doesn’t run out of water, the town council decides that, during any given

week, only half of the water remaining in the tank can be consumed. If the town does use exactly half of the water in the tank each week, how much water will be left after 8 weeks? **They will use 1,020 gallons which leaves 4 gallons.

Fibonacci Sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, …

Tell the story below:

Europe's first great medieval mathematician was the Italian Leonardo of Pisa, better known by his nickname Fibonacci. Although best known for the so-called Fibonacci Sequence of numbers, perhaps his most important contribution to European mathematics was his role in spreading the use of the Hindu-Arabic numeral system throughout Europe early in the 13th Century, which soon made the Roman numeral system obsolete, and opened the way for great advances in European mathematics.

Little is known of his life except that he was the son of a customs official and, as a child, he travelled around North Africa with his father, where he learned about Arabic mathematics. On his return to Italy, he helped to disseminate this knowledge throughout Europe, thus setting in motion rejuvenation in European mathematics, which had lain largely dormant for centuries during the Dark Ages.

In particular, in 1202, he wrote a hugely influential book called "Liber Abaci" ("Book of Calculation"), in which he promoted the use of the Hindu-Arabic numeral system, describing its many benefits for merchants and mathematicians alike over the clumsy system of Roman numerals then in use in Europe. Despite its obvious advantages, uptake of the system in Europe was slow (this was after all during the time of the Crusades against Islam, a time in which anything Arabic was viewed with great suspicion), and Arabic numerals were even banned in the city of Florence in 1299 on the pretext that they were easier to falsify than Roman numerals. However, common sense eventually prevailed and the new system was adopted throughout Europe by the 15th century, making the Roman system obsolete.

Fibonacci is best known, though, for his introduction into Europe of a particular number sequence, which has since become known as Fibonacci Numbers or the Fibonacci Sequence. He discovered the sequence - the first recursive number sequence known in Europe - while considering a practical problem in the "Liber Abaci" involving the growth of a hypothetical population of rabbits based on idealized assumptions. He noted that, after each monthly generation, the number of pairs of rabbits increased from 1 to 2 to 3 to 5 to 8 to 13, etc,

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The numbers of the sequence has also been found to be ubiquitous in nature: among other things, many species of flowering plants have numbers of petals in the Fibonacci Sequence; the spiral arrangements of pineapples occur in 5s and 8s, those of pinecones in 8s and 13s, and the seeds of sunflower heads in 21s, 34s, 55s or even higher terms in the sequence; etc.

In the 1750s, Robert Simson noted that the ratio of each term in the Fibonacci Sequence to the previous term approaches, with ever greater accuracy the higher the terms, a ratio of approximately 1 : 1.6180339887 (it is actually an irrational number equal to $(1 + \sqrt{5})/2$ which has since been calculated to thousands of decimal places). This value is referred to as the Golden Ratio, also known as the Golden Mean, Golden Section, Divine Proportion, etc, and is usually denoted by the Greek letter phi ϕ (or sometimes the capital letter Phi Φ). Essentially, two quantities are in the Golden Ratio if the ratio of the sum of the quantities to the larger quantity is equal to the ratio of the larger quantity to the smaller one. The Golden Ratio itself has many unique properties, such as $1/\phi = \phi - 1$ (0.618...) and $\phi 2 = \phi + 1$ (2.618...), and there are countless examples of it to be found both in nature and in the human world.

A rectangle with sides in the ratio of 1 : ϕ is known as a Golden Rectangle, and many artists and architects throughout history (dating back to ancient Egypt and Greece, but particularly popular in the Renaissance art of Leonardo da Vinci and his contemporaries) have proportioned their works approximately using the Golden Ratio and Golden Rectangles, which are widely considered to be innately aesthetically pleasing. An arc connecting opposite points of ever smaller nested Golden Rectangles forms a logarithmic spiral, known as a Golden Spiral. The Golden Ratio and Golden Spiral can also be found in a surprising number of instances in Nature, from shells to flowers to animal horns to human bodies to storm systems to complete galaxies.

8) Use the 7^{th} and 8^{th} terms of the Fibonacci sequence to approximate the golden ratio to three decimal places.
9) Find the lineage (number of bees in each generation), for seven generations, for a single, male bee if bees reproduce according to the following rules: If a female bee lays an egg and it does not get fertilized by a male, then the egg hatches as a male bee. If the female bee lays an egg and it is fertilized by a male bee, then the egg hatches as a female bee.
10) Using graph paper, construct a golden rectangle from the first 6 terms of the Fibonacci’s sequence.

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Homework #10

11) Using the first seven numbers from the Fibonacci sequence, construct a golden spiral. Use graph paper and tape together as many pieces as you deem necessary.