# Classroom Examples for Pre-Calculus #16

*The dimensions of a matrix are the # of rows by the # of columns (3x2)
*You can only add or subtract matrices with the same dimensions
*Discuss Scalar multiplication (single number) versus Dot Product (sum of the products)
*Matrix multiplication (Dot Product) – RC: like the soda…row times column
*Show them the trick of multiplying a 4x2 by a 2x3…you can only do it if the inside numbers match and the dimensions of the answer will be the outside numbers!

*Determinant (det(A) or $|A|$) – **Determinants are just a number!** determinants are used to solve systems of equations and to find inverses…for all square matrices, you place alternating plus and minus signs above the top row and in front of the first column. Cross out the first column and the top row. Multiply the determinant of the square matrix remaining by the number that was crossed out twice (whether you change the sign of this answer or keep it the same depends on whether the combination of the signs for the position of the double crossed out number makes a negative under the rules for multiplication…if the signs make a positive then you keep the sign of the answer…if the signs make a negative then you change the sign of the answer). Now cross out the next row and multiply the determinant of the square matrix remaining by the double crossed out number (you change the sign of this answer because there is a plus above that position and a minus sign in front it). Continue doing this until you have crossed out all of the rows of the matrix. *Keep in mind that the determinant of a 1x1 is just the number.* The determinant of the original matrix is the sum of all of these answers.

*Inverses ($A^{-1}$) – **Inverses are new matrices!** Inverses are used to find the identity matrix and to solve systems of equations. You create a matrix very similar to the one for a determinant by placing alternative plus and minus signs in front of the first column and above the top row. And, instead of only crossing out the first column with each row, you need to cross out every combination of column and row. You still find the determinant of the square matrix remaining but you do NOT multiply by the double crossed out number…you do still determine the signs of your answers by the rules of multiplication for the position of the double crossed out number just like in a determinant. The other difference is that you don't add up all of these answers. Instead, you replace each number in the original matrix with the answer you found from the rules above starting from the top left hand corner and working across until all positions have been filled. The final step is to multiply that entire matrix by the fraction $\frac{1}{det(A)}$.

For a 2x2 $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ do : $\frac{1}{det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

For a 3x3 $A=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ do: $\frac{1}{\det(A)}\begin{bmatrix} ei-hf & -(bi-hc) & bf-ec \\ -(di-gf) & (ai-gc) & -(af-dc) \\ dh-ge & -(ah-gb) & ae-db \end{bmatrix}$

Do a 4x4 by following the same rules and same patterns.

*Show how to set up and solve a 3x3 using matrices and inverses if there's time*

1) If possible, find $6B+2A$, $7A-4B$, $AB$, and $BA$ if $A=\begin{bmatrix} -9 & 0 \\ 3 & -5 \end{bmatrix}$ and $B=\begin{bmatrix} 0 & -7 \\ 4 & 1 \end{bmatrix}$

2) Find both the determinant and inverse, if it exists, of both $A=\begin{bmatrix} -5 & -9 \\ 4 & 6 \end{bmatrix}$ and $B=\begin{bmatrix} -2 & 5 \\ -7 & -12 \end{bmatrix}$

3) If possible, find $-6B$, $3A-B$, $AB$, and $BA$ if

$$A=\begin{bmatrix} -4 & 7 & 0 \\ 9 & -3 & 6 \\ 3 & -5 & 2 \end{bmatrix} \text{ and } B=\begin{bmatrix} 5 & 3 & 4 \\ -6 & -8 & -7 \\ 0 & -1 & -9 \end{bmatrix}$$

4) Find the determinant and, if it exists, the inverse of $A$, or $A^{-1}$, if $A=\begin{bmatrix} -3 & -4 & 6 \\ 2 & -1 & 5 \\ 3 & -2 & -6 \end{bmatrix}$ and

check your answers by finding $A^{-1}A$ and $AA^{-1}$

5) Find the determinant and, if it exists, the inverse of $A$, or $A^{-1}$, if $A=\begin{bmatrix} 3 & -5 & -2 & -3 \\ -2 & 4 & -1 & 5 \\ 4 & -3 & 3 & -2 \\ -1 & 1 & 2 & 1 \end{bmatrix}$ and

check your answers by finding $A^{-1}A$ and $AA^{-1}$