Classroom Examples for Pre-Calculus #17

*Solving systems using Cramer's rule versus using inverses
*McClaurin vs Cramer…1748 vs 1750…Cramer was more widely read…so he gets the name
*Cramer's rule uses the division of determinants to solve systems. The top determinant is created by replacing that column with the column on the right side of the equals sign. The bottom determinant is always formed from the column on the left side of the equals sign.
*Inverse method uses the inverse of the matrix on the left side to create the identity matrix by multiplying each side by the identity, A^{-1} has to come first in the multiplication…the order matters
*Decomposition of a Fraction into Partial Fractions – use long division if the power on top is greater than or equal to the bottom if you were to multiply out all of the pieces…then factor the bottom and use the following rules: just x means just A…x^2 means Ax+B

$$\frac{2x^6-4x^5+5x^4-3x^3+x^2+3x}{(x-1)^3(x^2+1)^2}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}+\frac{Dx+E}{x^2+1}+\frac{Fx+G}{(x^2+1)^2}$$

Then wipe out fractions and substitute for x to try to isolate certain variables…

1) Solve the following system of equations using matrices and the Cramer's rule:

$$3x+5y=-8$$
$$4x-6y=9$$

2) Decompose $\frac{3x^2+2x+8}{(x^2+3)(x-1)}$ into partial fractions.

3) Solve the following system of equations using matrices and the inverse:

$$6x-5y=2$$
$$3x-7y=-9$$

4) Decompose $\frac{-8x-17}{12x^2-7x-10}$ into partial fractions.

5) Solve the following system of equations using matrices and Cramer's rule:

$$5x-3y-4z=8$$
$$4x+2y+3z=-7$$
$$-3x+4y-z=9$$

6) Decompose $\frac{x^2+4x-1}{(x+3)^3}$ into partial fractions.

7) Solve the following system of equations using matrices and the inverse:

$$\begin{aligned} 3x - y + 4z &= -5 \\ x + 4y - 5z &= 8 \\ -2x - 3y + z &= -9 \end{aligned}$$

8) Decompose $\dfrac{x^2 + 3x - 5}{(x^2 - 13x + 30)(x + 4)}$ into partial fractions.

9) Solve the following problem using matrices and Cramer's rule. Emily's cookies are so good that she decides to go into business making cookies. She makes three kinds: chocolate chip, oatmeal, and sugar cookies. If she sells all the chocolate chip cookies for $2 per cookie, the oatmeal ones for $3 per cookie, and she sells the sugar cookies for $1 per cookie, she earns $88. Emily makes a total of 46 cookies. If five more than triple the difference between the amount of sugar cookies and oatmeal cookies is one less than the amount of chocolate chip cookies, how many of each kind did she make?

10) Decompose $\dfrac{5x^3 + 14x^2 - 7x}{x^2 + 6x - 8}$ into partial fractions.

11) Solve the following system of equations using matrices and the inverse:

$$\begin{aligned} 3w + 5x - 2y - z &= 12 \\ 2w - 4x + 3y + 5z &= -3 \\ -4w + 3x - y - 2z &= 11 \\ w - 2x + 4y + 3z &= -9 \end{aligned}$$