Classroom Examples for Pre-Calculus #18

*Discuss x/y graphs that turn into real/imaginary graphs
*Absolute value is the length of the hypotenuse in the right triangle formed
*Imaginary Land: Rectangular form versus polar form…rectangular is 1+ i …polar (trig form) is $\sqrt{2}(\cos 45^\circ + i\sin 45^\circ)$
*Remember: negative in front of the hypotenuse number means go in the opposite direction
All graphs on this worksheet are just points
*Benefits of polar form in Imaginary Land are the following rules:

Multiplication: $r_1(\cos\theta_1 + i\sin\theta_1)\cdot r_2(\cos\theta_2 + i\sin\theta_2) = r_1\cdot r_2[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$

Division: $\dfrac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \dfrac{r_1}{r_2}[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)]$

DeMoivre's Theorem: $[r(\cos\theta + i\sin\theta)]^n = r^n(\cos(n\theta) + i\sin(n\theta))$

Finding Roots: the nth roots of $r(\cos\theta + i\sin\theta) = r^{\frac{1}{n}}[\cos(\dfrac{\theta}{n} + k\dfrac{360^\circ}{n}) + i\sin(\dfrac{\theta}{n} + k\dfrac{360^\circ}{n})]$

*Where k = 0, 1, 2, 3…etc
**FYI – if you have to use have or double angle formulas that have +/- radicals in them, make sure that you check the quadrants because only one combination of signs will work!

1) Graph, find the absolute value, and convert $8\sqrt{3} - 8i$ into polar form.

2) Graph, find the absolute value, and convert $5(\cos 240^\circ + i\sin 240^\circ)$ into rectangular form.

3) Graph, find the absolute value, and convert 6 into polar form.

4) Graph, find the absolute value, and convert $-4(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3})$ into rectangular form.

5) Graph, find the absolute value, and convert $\dfrac{-7}{2} + \dfrac{7\sqrt{3}}{2}i$ into polar form.

6) Graph, find the absolute value, and convert $-6(\cos(\dfrac{-5\pi}{4}) + i\sin(\dfrac{-5\pi}{4}))$ into rectangular form.

7) Convert to polar form and then simplify $(2 + i\sqrt{3})(-1 + i)$

8) Convert to polar form and then simplify $\dfrac{5 - 5\sqrt{2}i}{\sqrt{3} + 2i}$

9) Simplify $(1-i\sqrt{3})^6$ and write your answer in polar form.

10) Simplify $(\frac{1}{2}-\frac{\sqrt{3}}{2}i)^8$ and write your answer in rectangular form.

11) Find (in polar form), and graph, the fifth roots of $32i$.

12) Find (in rectangular form), and graph, the fourth roots of -16.

13) Solve $x^5-243=0$ and put your answers in rectangular form.

14) Solve $x^6-2048\sqrt{3}+2048i=0$ and put your answers in polar form.

15) Solve $x^8+65536=0$ and put your answers in rectangular form.