Classroom Examples for Pre-Calculus #20

*Vectors – they have both magnitude and direction – special notation: Three forms:

1) Coordinate Component form: $u = \langle 1, \sqrt{3} \rangle$ 2) Polar form: $2\angle 60^{o}$ 3) Unit Vector Component form: $1i + \sqrt{3}j$ where i is the unit vector in the x direction $\langle 1,0 \rangle$ and j is the unit vector in the y direction $\langle 0,1 \rangle$

*The magnitude of a vector looks like absolute value: magnitude of vector u = $|u|$ and it is the length of the vector…the magnitude of two vectors is found from the sum of the vectors and it is the length of the hypotenuse of the right triangle formed.

*Review scalar multiplication $5\langle 4,-7 \rangle = \langle 20,-35 \rangle$

*Vector addition: $\langle -6,9 \rangle + \langle -4,3 \rangle = \langle -10,12 \rangle$

*Review Dot Product: $u \bullet v = |u| \cdot |v| \cdot \cos\theta$ and is calculated in component form as $x_u x_v + y_u y_v$ and when it is written like this: $\frac{u \bullet v}{|v|} = |u| \cdot \cos\theta$ it is the projection of u on v…when written like this: $\frac{u \bullet v}{|u|} = |v| \cdot \cos\theta$ it is the projection of v on u. **Demonstrate this with pictures showing shadows from the "sun" directly overhead…

*Discuss the importance of this for physics…for calculating total force in one direction…

*To find a vector's unit vector, divide the vector by the magnitude…

**Helpful hint: When finding the magnitude and direction of a the sum of two vectors, you must relate the final answer to one of the two original vectors: ex – u + v = 30 mag, 10 degrees from u

*Demonstrate how vector addition solves many problems already covered in this class and how vector subtraction solves many problems when you know the final answer and one component.

Explain how both definitions of a vector dot product help you find the angle between them

1) If $u = \langle 4,-5 \rangle$ and $v = \langle -6,3 \rangle$, find $7u - 5v$

2) If $u = \langle -2,9 \rangle$ and $v = \langle -4,-3 \rangle$, find $-6|-4u||5v|$

3) If $u = \langle -8,3 \rangle$ and $v = \langle 5,-7 \rangle$ and $w = \langle -4,-1 \rangle$, find $5u - 2v(4|w| - 3|u|)$

4) If $u = \langle -3,-11 \rangle$ and $v = \langle 8,-6 \rangle$ and $w = \langle -5,12 \rangle$, find $v \bullet w - w \bullet u$

5) Find a unit vector that has the same direction as the vector $9i - 7j$.

6) Find a unit vector that has the same direction as the vector $4\langle -3,6 \rangle + 2\langle 7,-1 \rangle$.

7) Determine the magnitude and direction angle, rounded to two decimal places, for the vector $\langle -6,13 \rangle$.

8) Determine the magnitude and direction angle, rounded to one decimal place, for the vector $5(6i+3j)-6(4i-9j)$.

9) If $|u|=43$ and $|v|=29$ and the angle in between the vectors, θ, equals 142°, find, rounded to two decimal places, the magnitude and direction of the vector $u+v$.

10) If $|u|=17$ and $|v|=31$ and the angle in between the vectors, θ, equals 51°, find, rounded to two decimal places, the magnitude and direction of the vector $u+v$.

11) Use vector addition to solve the following problem. A ship leaves a port and sails at a bearing of $N32^{\circ}W$ for 134 nautical miles. It then turns and sails for another 227 nautical miles at a bearing of $S47^{\circ}E$. Rounded to two decimal places, how far is the ship from the port and at what bearing?

12) Use vector addition to solve the following problem. An airplane takes off from an airport and flies at a bearing of 242° for 645 miles. It then turns and flies for another 509 miles at a bearing of 124°. Rounded to two decimal places, how far is the airplane from the airport and at what bearing?

13) Rounded to one decimal place, find the angle between the vectors $7i-6j$ and $-8i+5j$.

14) Use vector addition and the $i+j$ component form to solve the following problem. A motorboat's engine propels the boat with a 347 pound force in a direction of $N61^{\circ}E$. A current in the water coming from $S13^{\circ}W$ exerts a 97 pound force on the boat. Rounded to two decimal places, what is the magnitude of the resultant force and in what direction is the boat actually moving?

15) Use vector subtraction and the $i+j$ component form to solve the following problem. An airplane takes off from an airport with a groundspeed of 256 mph and needs to land at an airport bearing 296°. If there is a constant 47 mph wind blowing from 127°, what is the airspeed of the airplane and in what direction will the pilot have to actually fly the plane, rounded to two decimal places, in order to land at the appropriate airport?