Classroom Examples for Pre-Calculus #21

*Sequence is a list of numbers with a pattern *Series is the sum of a list of numbers with a pattern
*Arithmetic means constant increase or decrease using the same number: 5, 8, 11, 14…
*Geometric means increase or decrease by a constant multiplier: 2, 4, 8, 16, 32…

*Arithmetic Sequence: $a_{Last} = a_{termbeforethefirst} + d(x)$ Series: $Sum = x(a_1 + a_{Last}) \div 2$

*Geometric Sequence: $a_n = a_1 g r^{n-1}$ Series: $S_n = \frac{a_1(1-r^n)}{1-r}$

*Discuss the meaning of summation notation: $\sum_{k=1}^{5} 2^k = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 = 62$

*Both series equations only apply to finite sequences…finite series end
*Infinite Arithmetic series always result in an answer of positive or negative infinity
*Infinite Geometric series always result in an answer of positive or negative infinity unless $|r| < 1$ and then the sum is given by $S_\infty = \frac{a_1}{1-r}$ this also works for infinite summations $\sum_{k=1}^{\infty} ar^{k-1}$ because the first term will be a1 and r will be r!

*Helpful tip – if the summation starts with k = 0 then add one to the total amount of terms

*Bouncing ball word problem tips – Make a quick diagram!

Infinite 1) Write out the first few terms and see the pattern. 2) Each bounce results in double distance added. *Double the initial drop but remember that you must subtract that extra drop from your final answer later* 3) That makes a summation where the constant, a, equals twice the height of the initial drop and the r is the fractional height of each bounce: $\sum_{k=1}^{\infty} ar^{k-1}$ 4) Use the infinite geometric sum formula by using a as a1 and r as r and then subtract that extra drop you added in the beginning!

Finite 1) Write out the first few terms to see the pattern. 2) Each bounce results in double the distance. *Double the initial drop but remember that you must subtract that extra drop from your final answer later * 3) That makes a summation where the constant, a, equals twice the height of the initial bounce and the r is the fractional height of each bounce: $\sum_{k=1}^{\#ofbounces} ar^{k-1}$ 4) Use the finite geometric sum formula by using a as a1 and r as r and then subtract the extra drop you added in the beginning! 5) Fix the ending of the problem in one of two ways…if the ball is caught at its highest point after the final bounce, then add the **initial drop times r^# of bounces**…if the ball is stopped just as it hits the ground before making an extra bounce, then add twice that to the final answer!

1) Determine if the sequence -39, -22, -5, 12,… is arithmetic or geometric. Find an equation that models this sequence. Find the 34^{th} term and determine which term the number 879 represents.

2) Determine if the sequence $57344, 28672, 14336, 7168, \ldots$ is arithmetic or geometric. Find an equation that models this sequence. Find the 12^{th} term and determine which term the number $\frac{7}{8192}$ represents.

3) Evaluate: $\sum_{k=1}^{53} 4k - 78$

4) Evaluate: $\sum_{k=1}^{\infty} -3\left(\frac{-2}{3}\right)^k$

5) There are 13 seats in the first row of seats in an auditorium and each row behind that row contains 4 more seats than the previous row. If there are 45 rows of seats in this auditorium, determine the total number of seats.

6) Determine if the sequence $\frac{-3645}{4096}, \frac{-1215}{1024}, \frac{-405}{256}, \frac{-135}{64}, \cdots$ is arithmetic or geometric. Find an equation that models this sequence. Find the 11^{th} term and determine which term the number $\frac{-1310720}{19683}$ represents.

7) Determine if the sequence –381, –358, –335, –312,… is arithmetic or geometric. Find an equation that models this sequence. Find the 68^{th} term and determine which term the number 2,448 represents.

8) Evaluate: $\sum_{k=0}^{63} \frac{-3}{8}k + 13$

9) Evaluate: $\sum_{k=0}^{83} -6\left(\frac{10}{7}\right)^k$ (Write your final answer in scientific notation rounded to 6 places)

10) If a rubber ball is dropped from a height of ten feet and bounces straight back up to three-fourths of its initial height before falling back to the ground again and then repeats this pattern of bouncing up to three-fourths of its previous height and falling back down again forever, how far will the ball travel, in feet, exactly? Answer: 70ft

11) Nina's father makes 63 cannolis this week and sells them all. His cannolis are so good that he needs to make 14 more cannolis the next week to meet demand. If this pattern of demand continues for an entire year, how many cannolis will he have to make?

12) Determine if the sequence 52488, –34992, 23328, –15552,… is arithmetic or geometric. Find an equation that models this sequence. Find the 18^{th} term and determine which term the number $\frac{8192}{9}$ represents.

13) Evaluate: $\sum_{k=1}^{\infty} \frac{-7}{8}k+13$

14) Evaluate: $\sum_{k=0}^{\infty} -2(\frac{-5}{8})^k$

15) If a rubber ball is dropped from a height of eight feet and bounces straight back up to five sixths of its initial height before falling back to the ground again and then repeats this pattern of bouncing up to five sixths of its previous height and falling back down again for exactly eighteen bounces and then you grab it as it reaches its maximum height after the eighteen bounce, how far will the ball travel, in feet, rounded to four decimal places?