Classroom Examples for Pre-Calculus #22

*Divergent Series – A series that becomes infinite, either positive or negative
*Convergent Series – A series that gets infinitely close to a certain value as the number of terms approach infinity
*Harmonic Sequences and Series – named after the harmonics, or overtones, of full notes in music…the overtones of full wavelength notes occur at 1/2, 1/3, 1/4, 1/5, 1/6 wavelengths etc. Overtones make chords: C Note: wavelength = 20.812 m, ½ is C, 1/3 is G, ¼ is C, 1/5 is E
D Note: wavelength = 18.54 m, ½ is D, 1/3 is A, ¼ is D, and 1/5 is F#
**Prove that a harmonic series diverges – 1st step : 1+½+½+½+½+…goes to infinity
2nd step : 1+1/2+1/3+1/4+1/5+1/6+1/7+1/8+1/9…match the pieces so that every term of the harmonic is greater than or equal to each term in the in the ½ series by breaking up each ½ after the first one into fractions that equal 1/2. 1+1/2+1/4+1/4+1/8+1/8+1/8+1/8+1/16+…
**But alternating harmonic series always converge - $\sum_{k=1}^{\infty}\frac{(-1)^k}{k} = -1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}...$
**Show that an alternating harmonic converges to ln2 : $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k} = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...$
**Show that an alternating harmonic with odd numbers converges to $\frac{\pi}{4}$: $\sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1} = 1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+...$
*Quadratic Sequences and Series : $\sum_{k=1}^{n} k^2 = 1+4+9+16+25+...\quad sum = \frac{n(n+1)(2n+1)}{6}$
*Cubic Sequences and Series : $\sum_{k=1}^{n} k^3 = 1+8+27+64+125+...\quad sum = \frac{n^2(n+1)^2}{4}$
***The sum formulas for a quadratic or cubic only applies if the first term is 1.
***Explain why probability or expected value for yearly record rainfall is a harmonic series
***Explain why the crossing the desert in a fleet of jeeps is a normal or odd harmonic series
***Discuss strategies for creating harmonic, quadratic or cubic formulas from patterns

1) Determine if the series 289 + 361 + 400 + 441 + … is arithmetic, geometric, harmonic, quadratic, or cubic. Find a formula that models this series. Find the sum of the first 35 terms of this series without actually writing them all down and adding up the results.

2) What kind of series is $\sum_{k=1}^{\infty}\frac{1}{2k+1}$ and state whether it converges or diverges. Write out the first seven terms of the sequence and find their sum exactly AND rounded to five decimal places. **Repeat same question with $\sum_{k=1}^{\infty}\frac{(-1)^k}{k+1}$

3) Use the first eleven terms of the harmonic sequence used to calculate π to approximate π to five decimal places.

4) The following represents all of the wavelengths of all of the musical notes for four octaves, rounded to one decimal place:

C note (20.8, 10.4, 5.2, 2.6), D note (18.5, 9.3, 4.6, 2.3), E note (16.5, 8.3, 4.1, 2.1), F note (15.6, 7.8, 3.9, 1.9), G note (13.9, 6.9, 3.5, 1.7), A note (12.4, 6.2, 3.1, 1.5), B note (11.0, 5.5, 2.8, 1.4), C# note (9.8, 4.9, 2.5, 1.2), D# note (17.5, 8.8, 4.4, 2.2), F# note (14.7, 7.4, 3.7, 1.8), G# note (13.1, 6.6, 3.3, 1.6), and A# note (11.7, 5.8, 2,9, 1.5)

Use this information along with your knowledge of harmonic series to determine and prove the three notes in a C chord. ***Repeat as necessary asking for D, F, or G chords!***

5) Use the first nine terms of the harmonic sequence used to calculate $\ln 2$ to approximate $\ln 2$ to four decimal places.

6) Determine if the series 729 + 1000 + 1331 + 2744 + … is arithmetic, geometric, harmonic, quadratic, or cubic. Find a formula that models this series. Find the sum of the first 15 terms of this series without actually writing them all down and adding up the results.

7) If you start keeping track of the yearly rainfall in your particular location on the first of this year, what is the expected value of record-breaking yearly rainfalls in the next 5 years? What would the expected value be for the next 15 years? (round your answers to the nearest whole numbers) ***Use TI-89 to demonstrate 100, 500, and 1000 year answers***

8) What kind of sequence is $1, -\frac{1}{4}, \frac{1}{7}, -\frac{1}{10}, \ldots$? Find a formula for the sum of that sequence and state whether the series converges or diverges. Find the sum of the first seven terms of the sequence exactly AND rounded to five decimal places.

9) What kind of sequence is $\frac{1}{3}, \frac{1}{7}, \frac{1}{11}, \frac{1}{15}, \ldots$? Find a formula for the sum of that sequence and state whether the series converges or diverges. Find the sum of the first eight terms of the sequence and exactly AND rounded to five decimal places.

10) Your goal is to cross a vast desert that is 479 miles wide. You own a huge fleet of jeeps, with drivers, so you can use as many jeeps as needed to help get you across the desert. There are no gas stations or refueling options anywhere in this desert and, for safety reasons, the only gas you can bring with you is what will fit in the gas tank of each jeep. If each jeep can go 219 miles on a tankful, how many jeeps will it take for you to cross the desert safely? **Repeat this question but with the condition that each jeep can carry one empty gas can and that no jeeps or drivers can be left stranded in the desert!** ***First answer is 1+1/2+1/3+1/4… while second answer is 1+1/3+1/5+1/7…