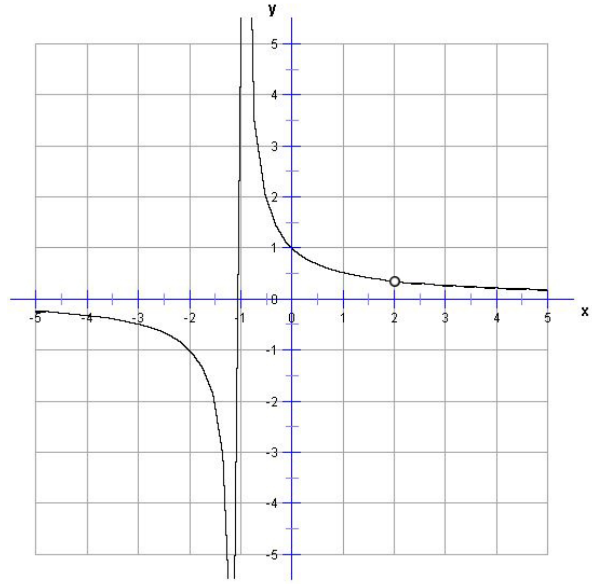
Calculus – Homework #1 - Answer Key

1)

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	.3448	.3344	.3334	.3332	.3322	.3226

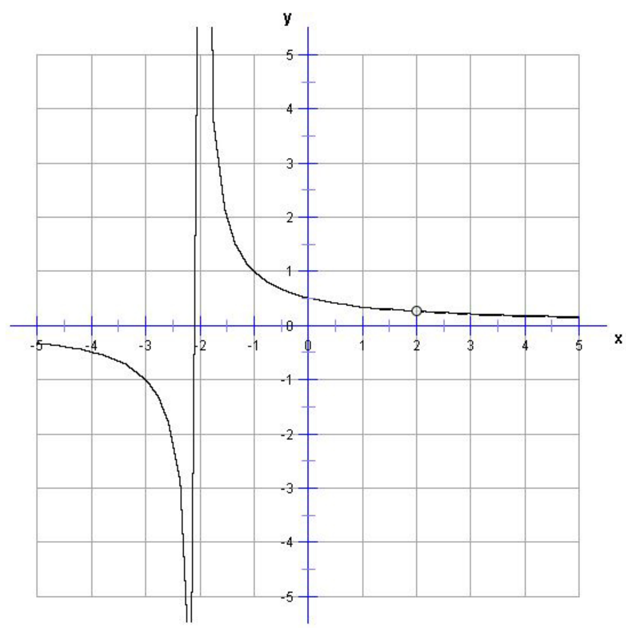
$$\lim_{x\to 2}\frac{x-2}{x^2-x-2}=0.\bar{3} \text{ or } \frac{1}{3}$$



2)

x	-1.9	-1.99	-1.999	-2.001	-2.01	-2.1
f(x)	10	100	1000	-1000	-100	-10

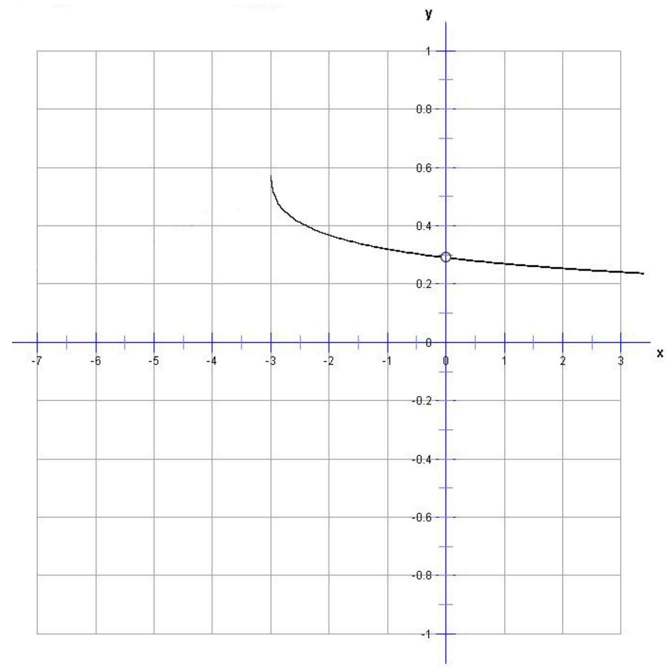
$$\lim_{x\to -2}\frac{x-2}{x^2-4}= \text{ Does Not Exist}$$



3)

x	.1	.01	.001	-.001	-.01	-.1
f(x)	.2863	.2884	.2887	.2887	.2889	.2911

$$\lim_{x\to 0}\frac{\sqrt{x+3}-\sqrt{3}}{x}\approx .2887$$

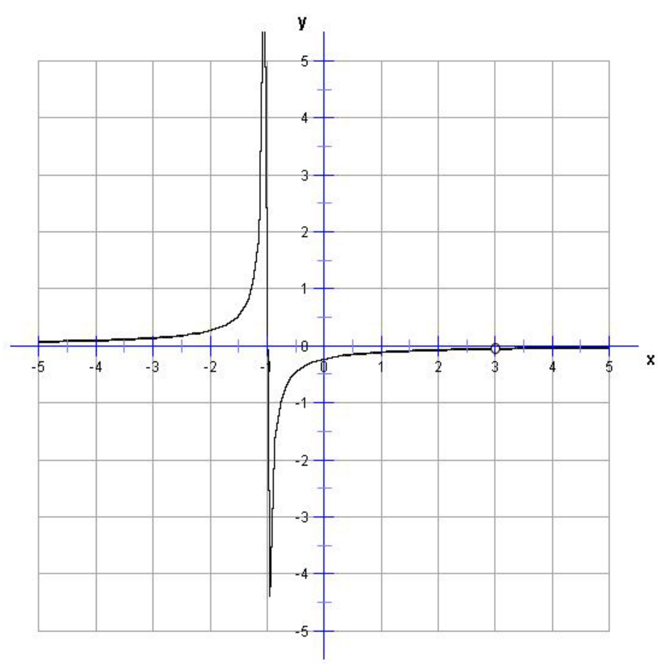


4) $L=5$ Proof: Find a relationship between ε and δ by stating that $0<|x-a|<\delta$ where $a=2$ which gives you $|x-2|<\delta$ by substitution. Now state that $|f(x)-L|<\varepsilon$. Substitution yields $|(x+3)-(5)|<\varepsilon$. Simplify to get $|x-2|<\varepsilon$ which means that the relationship between δ and ε is $\delta=\varepsilon$. Therefore, if $0<|x-2|<\delta$, then $|(x-2)|<\varepsilon$ which means $|(x+3)-(5)|<\varepsilon$

5)

x	3.1	3.01	3.001	2.999	2.99	2.9
f(x)	-.0610	-.0623	-.0625	-.0625	-.0627	-.0641

$$\lim_{x\to 3}\frac{\frac{1}{x+1}-\frac{1}{4}}{x-3}=-.0625 \text{ or } \frac{-1}{16}$$

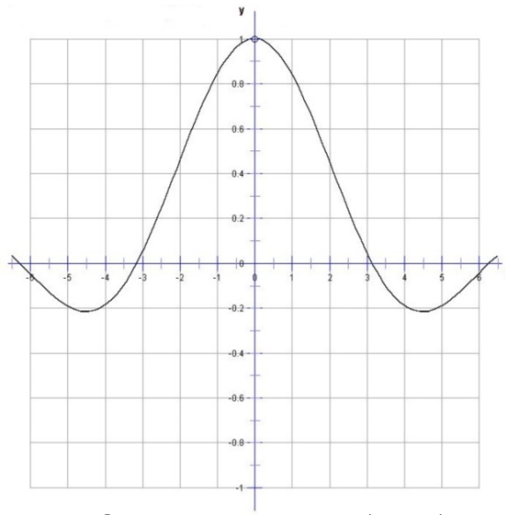


6) $L = 8$ $\delta = .00\bar{3}$ or $\dfrac{1}{300}$

7)

x	.1	.01	.001	-.001	-.01	-.1
f(x)	.9983	1.0000	1.0000	1.0000	1.0000	.9983

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

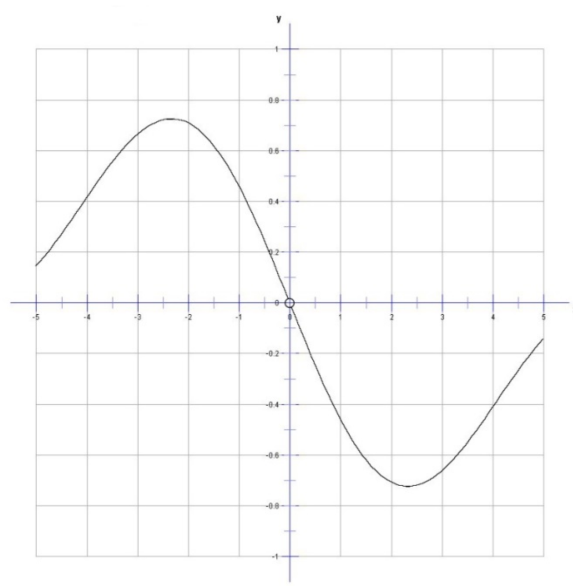


8) $L = -8$ Proof: Find a relationship between ε and δ by stating that $0 < |x-a| < \delta$ where $a = 4$ which gives you $|x-4| < \delta$ by substitution. Now state that $|f(x) - L| < \varepsilon$. Substitution yields $|(-3x+4)-(-8)| < \varepsilon$. Simplify to get $|-3x+12| < \varepsilon$ you must now manipulate this inequality to get $|-3(x-4)| < \varepsilon$ which is equivalent to $3|(x-4)| < \varepsilon$ and finally $|x-4| < \dfrac{\varepsilon}{3}$. This means that the relationship between δ and ε is $\delta = \dfrac{\varepsilon}{3}$. Therefore, if $0 < |x-4| < \delta$, then $0 < |x-4| < \dfrac{\varepsilon}{3}$ then $3|x-4| < \varepsilon$ then $|-3(x-4)| < \varepsilon$ which can then be written as $|-3x+12| < \varepsilon$ or $|(-3x+4)+8| < \varepsilon$ and finally as $|(-3x+4)-(-8)| < \varepsilon$

9)

x	.1	.01	.001	-.001	-.01	-.1
f(x)	-.0500	-.0050	-.0005	.0005	.0050	.0500

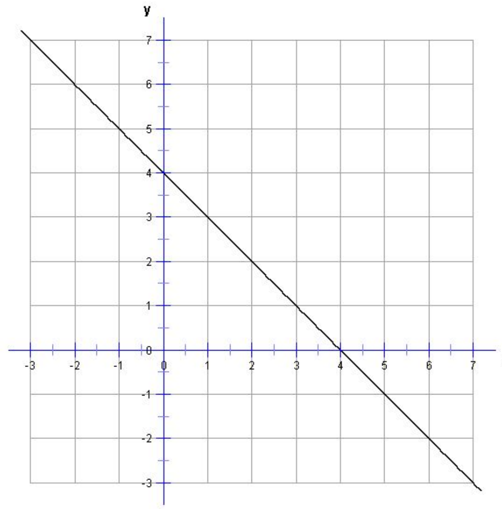
$$\lim_{x\to 0}\frac{\cos x - 1}{x} = 0$$



10)

x	3.1	3.01	3.001	2.999	2.99	2.9
f(x)	.9000	.9900	.9990	1.0010	1.0100	1.1000

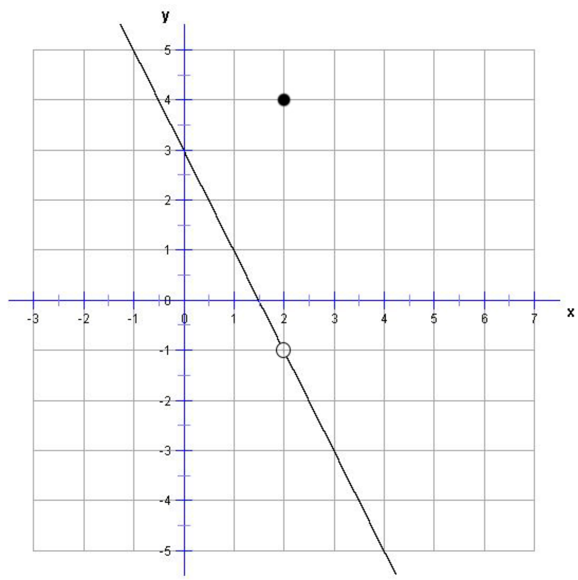
$$\lim_{x\to 3}(4-x)=1$$



11)

x	2.1	2.01	2.001	1.999	1.99	1.9
f(x)	-1.2000	-1.0200	-1.0020	-.9980	-.9800	-.8000

$$\lim_{x\to 2}f(x)=-1$$



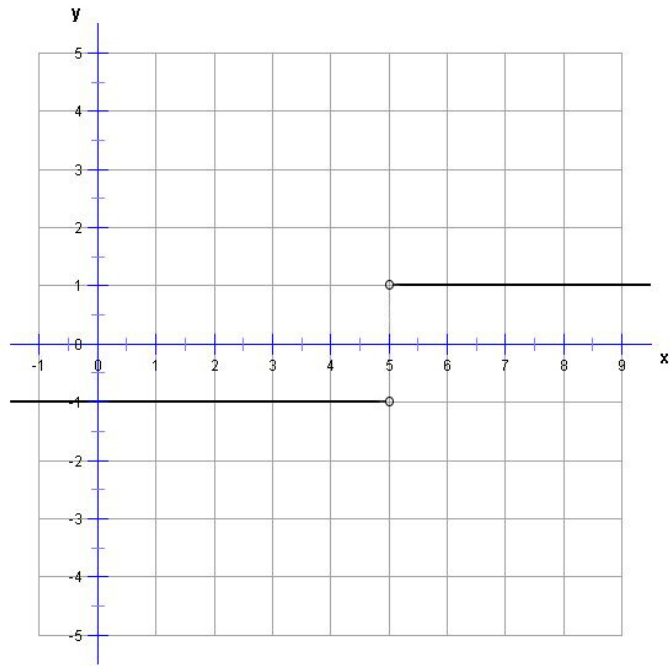
12) $L=0$ Proof: Find a relationship between ε and δ by stating that $0<|x-a|<\delta$ where $a=0$ which gives you $|x-0|<\delta$ by substitution. Now state that $|f(x)-L|<\varepsilon$. Substitution yields $\left|\sqrt[3]{x}-0\right|<\varepsilon$. Simplify to get $\left|\sqrt[3]{x}\right|<\varepsilon$ you must now manipulate this inequality to get $|x|<\varepsilon^3$ which is equivalent to $|x-0|<\varepsilon^3$. This means that the relationship between δ and ε is $\delta=\varepsilon^3$. Therefore, if $0<|x-0|<\delta$, then $0<|x-0|<\varepsilon^3$ which can become $\sqrt[3]{|x-0|}<\varepsilon$ which simplifies to $\left|\sqrt[3]{x-0}\right|<\varepsilon$ which can then be written as $\left|\sqrt[3]{x}-0\right|<\varepsilon$.

13) $L = 1$ $\delta = .002$ or $\dfrac{1}{500}$

14)

x	5.1	5.01	5.001	4.999	4.99	4.9
f(x)	1.0000	1.0000	1.0000	-1.0000	-1.0000	-1.0000

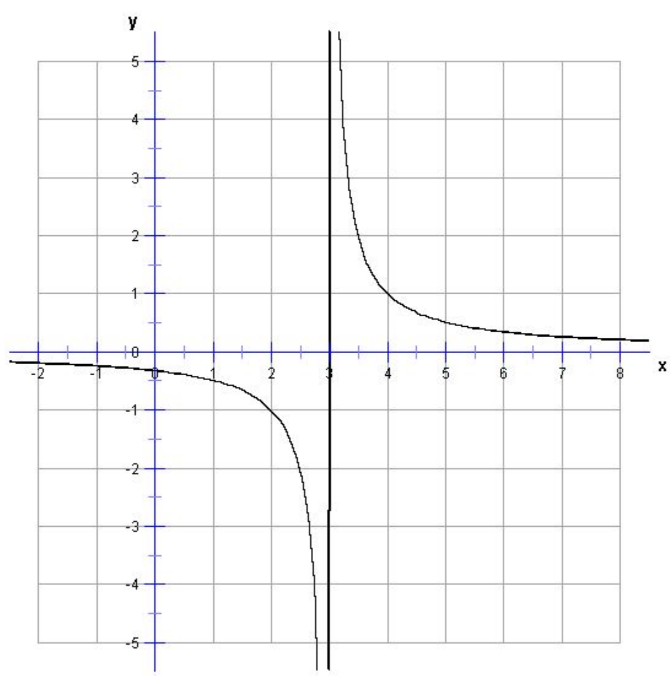
$$\lim_{x \to 5} \frac{|x-5|}{x-5} = \text{Does Not Exist}$$



15)

x	3.1	3.01	3.001	2.999	2.99	2.9
f(x)	10.0000	100.0000	1000.0000	-1000.0000	-100.0000	-10.0000

$$\lim_{x \to 3} \frac{1}{x-3} = \text{Does Not Exist}$$

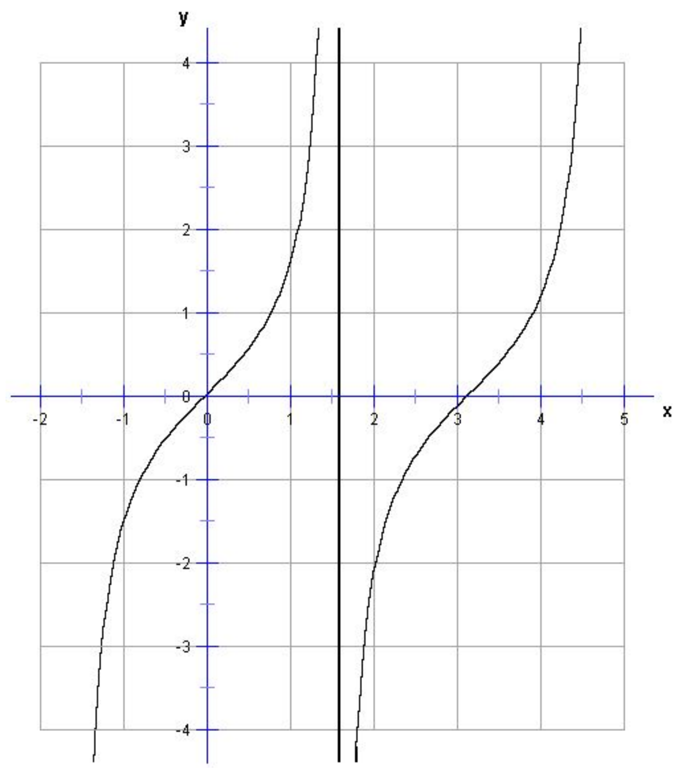


16) $L = 2$ Proof: Find a relationship between ε and δ by stating that $0 < |x - a| < \delta$ where $a = 1$ which gives you $|x - 1| < \delta$ by substitution. Now state that $|f(x) - L| < \varepsilon$. Substitution yields $|(x^2 + 1) - (2)| < \varepsilon$. Simplify to get $|x^2 - 1| < \varepsilon$ you must now manipulate this inequality to get $|(x+1)(x-1)| < \varepsilon$. In a case like this, the only way to produce $x - 1$ inside the absolute value is to set a boundary on the proof by making x equal something close to a. If $x = 2$, then the proof is valid for all values of x as long as $a < x \le 2$. By substitution, $|(x+1)(x-1)| < \varepsilon$ becomes $|3(x-1)| < \varepsilon$ which is equivalent to $3|(x-1)| < \varepsilon$ and finally $|(x-1)| < \frac{\varepsilon}{3}$. This means that the relationship between δ and ε is $\delta = \frac{\varepsilon}{3}$. Therefore, if $0 < |x-1| < \delta$, then $0 < |x-1| < \frac{\varepsilon}{3}$ then $3|x-1| < \varepsilon$ then $|3(x-1)| < \varepsilon$ which can then be written as $|(x+1)(x-1)| < \varepsilon$ if $a < x \le 2$ where the maximum value of x is 2. This can be written as $|x^2 - 1| < \varepsilon$ or $|x^2 + 1 - 2| < \varepsilon$ and finally as $|(x^2 + 1) - (2)| < \varepsilon$

17)

x	1.6708	1.5808	1.5718	1.5698	1.5608	1.4708
f(x)	-9.9663	-99.9600	-996.3400	1003.6864	100.0334	9.9670

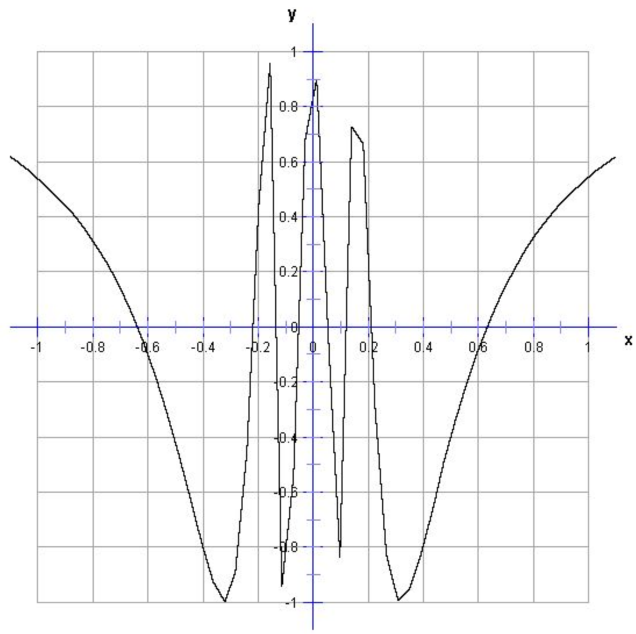
$$\lim_{x \to \frac{\pi}{2}} \tan x = \text{Does Not Exist}$$



18)

x	.1	.01	.001	-.001	-.01	-.1
f(x)	-.8391	.8623	.5624	.5624	.8623	-.8391

$$\lim_{x \to 0} \cos\left(\frac{1}{x}\right) = \text{Does Not Exist}$$



19) $L = 2$ $\delta = .02$ or $\frac{1}{50}$

20) $L = -1$ Proof: Find a relationship between ε and δ by stating that $0 < |x - a| < \delta$ where $a = -3$ which gives you $|x - (-3)| < \delta$ by substitution and simplify to $|x + 3| < \delta$. Now state that $|f(x) - L| < \varepsilon$. Substitution yields $|(2x + 5) - (-1)| < \varepsilon$. Simplify to get $|2x + 6| < \varepsilon$ you must now manipulate this inequality to get $|2(x + 3)| < \varepsilon$ which is equivalent to $2|(x + 3)| < \varepsilon$ and finally produces $|(x + 3)| < \frac{\varepsilon}{2}$. This means that the relationship between δ and ε is $\delta = \frac{\varepsilon}{2}$. Therefore, if $0 < |x + 3| < \delta$, then $0 < |x + 3| < \frac{\varepsilon}{2}$ then $2|x + 3| < \varepsilon$ then $|2(x + 3)| < \varepsilon$ which can be simplified to $|2x + 6| < \varepsilon$ which can become $|2x + 5 + 1| < \varepsilon$ and finally as $|(2x + 5) - (-1)| < \varepsilon$