Teaching Notes for Calculus
Homework #7
The Chain Rule and More Trig Rules for Derivatives

The Chain Rule is the fact that $\frac{dy}{dx}$ can be written as $\frac{dy}{du} \cdot \frac{du}{dx}$. Writing a complex derivative in this way, one that consists of a function inside another function, is the same as doing a derivative on composite functions. For example, if you let $g(x) = u$ and $f(u) = y$ then $\frac{dy}{du} \cdot \frac{du}{dx} = f'(u) \cdot g'(x) = f'(u) \cdot u'$. Therefore, if you need to take a derivative of a function inside another function, you can use the formula $f'(u) \cdot u'$. Illustrate this concept with this example:

If $y = 6(-3x-2)^3$, find $\frac{dy}{dx}$:

Answer: Let $u = -3x - 2 \therefore \frac{du}{dx} = -3$ so $y = 6(u)^3 \therefore \frac{dy}{du} = 18u^2 \frac{du}{dx} \therefore \frac{dy}{dx} = -54(-3x-2)^2$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

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Classroom Examples:

1) Find $f'(x)$ if $f(x) = \sec(\csc x) - 2(3x-1)^2$

Answer: $f'(x) = -\csc x \cdot \cot x \cdot \sec(\csc x) \cdot \tan(\csc x) - 36x - 12$

2) Find $f'(x)$ if $f(x) = (4x^2 - 3x)^2(3x^3 + 2x^2)^2$

Answer:

$$f'(x) = 2(8x-3)(3x^3+2x^2)^2(4x^2-3x) + 2(4x^2-3x)^2(9x^2+4x)(3x^3+2x^2)$$
$$f'(x) = 1440x^9 - 216x^8 - 1144x^7 + 84x^6 + +216x^5$$

3) Find $\dfrac{dy}{dx}$ if $y = \sec\sqrt{x} - \sqrt{\tan x}$

Answer:

$$\frac{dy}{dx} = \frac{\sqrt{x}}{2x} \cdot \sec\sqrt{x} \cdot \tan\sqrt{x} - \frac{\sqrt{\tan(x)}}{2\tan(x)}\sec^2 x = \frac{\sqrt{x} \cdot \tan\sqrt{x}}{2x \cdot \cos\sqrt{x}} - \frac{\sqrt{\tan(x)}}{2\sin(x) \cdot \cos(x)}$$

4) Find the derivative of $f(x) = \dfrac{2}{x^2} - \sqrt{\csc x}$ at the point $\left(\dfrac{\pi}{2}, \dfrac{8-\pi}{\pi}\right)$

Answer: $f'(x) = \dfrac{-4}{x^3} - \dfrac{\csc x \cdot \cot x}{2 \cdot \sqrt{\csc x}}$ therefore $f'\left(\dfrac{\pi}{2}\right) = \dfrac{-32}{\pi^3}$

5) Find $\dfrac{dy}{dx}$ if $y = -x^3\sqrt{4-3x^2}$

Answer:

$$\frac{dy}{dx} = -3x^2 \cdot \sqrt{4-3x^2} + \frac{3x^4}{\sqrt{4-3x^2}} = -3x^2 \cdot \sqrt{4-3x^2} + \frac{3x^4\sqrt{4-3x^2}}{4-3x^2} = \frac{12x^2\sqrt{4-3x^2}\left(x^2-1\right)}{4-3x^2}$$

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6) Find the equation of the line tangent to $y = 3x^2\sqrt{2x^4+4}$ at the point $(-2, 72)$

Answer: $y' = 3x^2\sqrt{2x^4+4} = 6x\sqrt{2x^4+4} + \dfrac{12x^5}{\sqrt{2x^4+4}}$

$y'(-2) = 6(-2)\sqrt{2(-2)^4+4} + \dfrac{12(-2)^5}{\sqrt{2(-2)^4+4}} = -136 \quad y = -136x + b$

$(72) = -136(-2) + b \quad b = -200$ Therefore the tangent line is $y = -136x - 200$

7) Find the equation of the line tangent to $y = -\cot^2 x$ at the point $\left(\dfrac{7\pi}{6}, -3\right)$

Answer: $y' = 2\cdot\csc^2 x\cdot\cot x \quad y'\left(\dfrac{7\pi}{6}\right) = 2\cdot\csc^2\left(\dfrac{7\pi}{6}\right)\cdot\cot\left(\dfrac{7\pi}{6}\right) = 8\sqrt{3}$

$y = 8\sqrt{3}x + b \quad (-3) = 8\sqrt{3}(\dfrac{7\pi}{6}) + b \quad b = \dfrac{-28\pi\sqrt{3}}{3} - 3 = -\dfrac{28\pi\sqrt{3}+9}{3}$

Therefore the tangent line is $y = 8\sqrt{3}x - \dfrac{28\pi\sqrt{3}+9}{3}$